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ABSTRACT

This book contains selected research papers presented at seminars held throughout the year 2000 in Finland by members of the Finnish Association for Research in Mathematics and Science Education (FARMSE) and students at the Finnish Graduate School of Mathematics, Physics, and Chemistry Education. This volume also contains papers professor Laurence Viennot, Universite Paris VII, France and professor Georgios Tsaparlis, University of Ioannina, Greece, who gave lectures and workshops at the spring seminar of the Graduate School in Joensuu, Finland, and by professor Erikki Pehkonen, who gave a lecture at the summer seminar in Vaasa, Finland. Papers include: (1) "The Finnish Graduate School of Mathematics, Physics and Chemistry Education" (Maija Ahte and Virpi Vatanen); (2) "A Hidden Regulating Factor in Mathematics Classrooms: Mathematics-Related Beliefs" (Erkki Pehkonen); (3) "Primary School Teachers' Mathematics Beliefs, Teaching Practices and Use of Textbooks" (Paivi Perkkila); (4) "Mathematics for Primary School Teachers" (Silja Pesonen); (5) "The Metalevel of Cognition-Emotion Interaction" (Markku S. Hannula); (6) "Problem Solving in Chemistry and Science Education" (Georgios Tsaparlis); (7) "Physics Education Research: Inseparable Contents and Methods--The Part Played by Critical Details" (Laurence Viennot); (8) "The Force Concept Inventory in Diagnosing the Conceptual Understanding of Newtonian Mechanics in Finnish Upper Secondary Schools" (Johanna Jauhiainen, Ismo T. Koponen, and Jari Lavonen); and (9) "An Evaluation of Interactive Teaching Methods in Mechanics: Using the Force Concept Inventory To Monitor Student Learning" (Antti Savinainen). (Most papers contain references.) (YDS)

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RESEARCH ON MATHEMATICS AND SCIENCE EDUCATION

From Beliefs to Cognition, from Problem Solving to Understanding

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Research on Mathematics and Science Education

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From Beliefs to Cognition, from Problem Solving to Understanding

Maija Ahtee, Ole Björkqvist, Erkki Pehkonen
and Virpi Vatanen (Eds.)

INSTITUTE FOR EDUCATIONAL RESEARCH
UNIVERSITY OF JYVÄSKYLÄ

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The Finnish Association for Research in Mathematics and Science Education
The Finnish Graduate School of Mathematics, Physics and Chemistry Education

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Preface

The book at hand is a joint project of the Finnish Association for Research in Mathematics and Science Education (Professor Erkki Pehkonen chair), and the Finnish Graduate School of Mathematics, Physics and Chemistry Education (Professor Maija Ahtee chair). The papers were presented at the seminars during the year 2000. Research in mathematics and science education in Finland is relatively young. Since education is delivered in Finnish (and Swedish) and since one of the main aims of doing research in education is to find ways of developing teaching and learning in the schools, most of the theses are written in Finnish. It is also important that teachers read about research findings.

However, one of the most important ways of raising the quality of research is, of course, to get an international evaluation of the reports. At the meeting of the board members of the Graduate School in the autumn of 2000 the idea was brought up that we should offer the members of the Research Association as well as the Graduate School students the possibility to publish their presentations in English. Professors Maija Ahtee (University of Jyväskylä), Ole Björkqvist (Åbo Akademi University) and Erkki Pehkonen (University of Turku) were chosen as the board of editors. Before papers were accepted for publication, they were to pass a careful referee process.

The referees of the book are Ann Ahlberg (University of Gothenburg, Sweden), Otto Bekken (University of Kristiansand, Norway), Harrie Eijkelhof (University of Utrecht, the Netherlands), Fulvia Furinghetti (University of Genua, Italy), Richard Gott (University of Durham, UK), Barbro Grevholm (University of Kristianstad, Sweden), Erika Kuendiger (University of Windsor, Canada), Sinikka Lindgren (University of Tampere, Finland), Thomas Lingefjärd (University of Gothenburg, Sweden), Jukka Maalampi (University of Jyväskylä, Finland), Olof Magne (University of Lund, Sweden), Robin Millar (University of York, UK), George Philippou (University of Cyprus), Phil Scott (University of Leeds, UK), Jouni Viiri (University of Joensuu, Finland), Mikko Vuolle (University of Jyväskylä, Finland). Our referees were very strict but also helpful in case they thought the papers were sound and worth publishing. We want to express our sincere thanks to all of them for their very thorough work.

This volume also contains papers by professor Laurence Viennot, Universite Paris VII, France, and professor Georgios Tsaparlis, University of Ioannina, Greece, who gave lectures and workshops at the spring seminar of the Graduate School in Joensuu, and by professor Erkki Pehkonen, who gave a lecture at the summer seminar in Vaasa.

During the time period 1996-2001, the Finnish Graduate School was supported financially by the Ministry of Education. The activities of the Graduate School greatly promoted research done within mathematics and science education. Today we are working to find other ways of raising the standard of research within these fields in Finland.

Finally, we want to express our sincere thanks to professor Jouni Välijärvi, Institute for Educational Research, University of Jyväskylä. Without his support we would not have been able to publish this volume.

The Editors

The Finnish Graduate School of Mathematics, Physics and Chemistry Education

Maija Ahtee and Virpi Vatanen
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The new graduate school system was started by the Academy of Finland in 1995 to promote the quality and effectiveness of the post graduate education. In this connection also a graduate school concentrating on educational problems in the fields of mathematics, physics and chemistry was founded. Earlier only a few dissertations in the field of science education had been presented and the doctorates had been working more or less alone with their research problems. In this article, the principles and activities of the graduate school are described. Regrettably, the Academy of Finland did not value high enough the work done in this graduate school in past five years and so the funding of the graduate school will finish in the end of year 2001.

The rise of the graduate school

The Graduate School System to promote the quality and effectiveness in graduate students' education especially through the co-operation of the universities was initiated by the Academy of Finland in 1995. For example in science the training of the post graduates consisted earlier mostly of some theoretical studies and research work in small research groups in the specific departments. The aims of the new Graduate School System were to decrease the average age of the new doctors (37 years) and to increase the co-operation of the research groups both nationally and internationally. At the same time the old research positions in the Academy for post-graduates were suspended.

As the Academy of Finland implied in the announcement that also new types of graduate schools should be suggested an initiative was made to find out the possibility to start a national graduate school for active mathematics, physics or chemistry teachers who wanted to do research on educational problems. A preliminary effort had already been made in the Department of Teacher Education at the University of Helsinki in autumn 1994 when Maija Ahtee and Erkki Pehkonen had started a post graduate course in mathematics, physics and chemistry education. Approximately fifty teachers throughout Finland expressed their interest to take part in this course.

After the first unsuccessful attempt the representatives from the departments of mathematics, physics, chemistry and teacher education of the five universities, Helsinki, Joensuu, Jyväskylä, Oulu and Åbo Academy were invited to the Department of Teacher Education at Helsinki University. The goal of the meeting was to discuss the practical issues related to a Graduate School in the field of mathematics, physics and chemistry education. The participants were all in favour of the project. One representative from each university was selected to the board of the Graduate School. Professor Kaarle Kurki-Suonio from the Department of Physics at Helsinki University agreed to act as the director of the Graduate School. In spring 1995 the Academy of Finland provided funding for a four-year period to four full time students starting from September 1995.

From the beginning of 1998 the Graduate School expanded when the Ministry of Education provided funding for ten full-time researchers for a four-year period. The expansion was not entirely complete since at the same time the earlier four vacancies were terminated. The universities of Turku and Lapland also joined the Graduate School, thus increasing the number of member universities to seven and the number of departments to twenty-three.

The aim and activities of the Graduate School

The aim of the Graduate School is to create a solid, research-based foundation for the continuous development of mathematics, physics and chemistry education and educational research in Finland, both at school and university level. To ensure the standard in both the pedagogical and subject matter knowledge two supervisors are appointed for each postgraduate student, one representing educational expertise and the other scientific expertise. Research combines content and methodological perspectives from mathematics, physics, chemistry, and education and serves the structural, content-related and methodological development of teaching. The study areas of the Graduate School consist of research and development of mathematics, physics and chemistry education from pre-school to tertiary level. The current main research topics include for example development of teaching in secondary school, research on empirical methods, research on mathematics and science learning, beliefs and concepts, multimedia-aided teaching, and primary school mathematics and science teaching.

The Graduate School arranges two or three seminars annually in each of the member universities in turn. Graduate students present their study-plans to the supervisors and the students for evaluation. The summer seminar is intensive one-week period concentrating on research methods, while the autumn seminar is arranged in collaboration with the Mathematics and Science Education Research Association. The plenary lecturers are well known international researchers in the field of mathematics and science education. Each university also has its own local postgraduate study group.

The Graduate School offers full time researchers an opportunity to concentrate on studies and research and supports their participation in international and national conferences: it also offers co-operation in the scope of the Graduate School and maintains an information channel between the students and the supervisors.

The principles of the Graduate School

Admission to the postgraduate program requires advanced studies in the major subject and a personal study-plan for the completion of a licentiate or doctoral degree. The study-plan should contain equal and meaningful amounts of both pedagogical and subject studies. The student must be a qualified teacher and preferably also have some experience in teaching. Subject studies in mathematics, physics or chemistry are required if the student has obtained his/her Master's degree in education. Permitting the licentiate degree as an aim for postgraduate studies is based on the fact that it is a formal requirement for becoming a lecturer in polytechnics or in departments of teacher education in the universities.

At the beginning of 2001 the Graduate School had a total of 107 students. Ten full-time students were funded by the Ministry of Education and four students by universities or foundations. There were 93 part-time students who pursued their studies alongside teaching. The Graduate School had 54 supervisors in the member universities.

The results

During the past five years, ten didactically oriented doctoral dissertations have been completed; three in physics, one in chemistry, and six in education, where two dealt with physics, two with chemistry, and two with mathematics teaching. Furthermore, 25 licentiate degrees have been completed. The majority of the degrees are completed at the Universities of Helsinki and Joensuu, which together are responsible for eight doctoral and 17 licentiate degrees. The amount of postgraduate degrees has increased in 2000; half of the dissertations and 40% of the licentiate examinations were completed last year.

The publications of the graduate students include refereed articles in scientific journals, proceedings and in congress abstracts, scientific theses, and textbooks for schools and educational columns in the national media. Approximately 35 publications have been produced each year during the Graduate School's existence (see Table 1).

TABLE 1. Publications of the graduate students in all the member universities between 1.9.1995 – 31.8.2000.

Year	Helsinki Univ.	Joensuu Univ.	Jyväskylä Univ.	Oulu Univ.	Turku Univ.	Åbo Academy	Total
1995 + 1996	23	4	-	-	-	1	28
1997	19	1	1	3	-	1	25
1998	20	13	3	4	-	6	46
1999	18	10	10	2	3	2	45
2000	4	9	11	3	4	1	32
Total	84	37	25	12	7	11	Avg. 35.2

The Graduate School encourages students to participate in international conferences and to carry out postgraduate studies abroad. For this activity a grant of 800 EURO for travel expenses is reserved for each student. The prerequisite for this is that the student presents his/her research results either in an oral or poster session at the conference. The students have taken ample advantage of this grant: in 2000 nine full-time students presented their results before an international forum. During the academic year 1999–2000 two students carried out postgraduate studies at a foreign university. In the past five years the students have had approximately 23 international contacts per year (see Table 2).

TABLE 2. International contacts of graduate students in all the member universities between 1.9.1995 – 31.8.2000.

Year	Helsinki Univ.	Joensuu Univ.	Jyväskylä Univ.	Oulu Univ.	Turku Univ.	Åbo Academy	Total
1995 + 1996	16	2	-	2	-	2	22
1997	11	3	1	2	-	10	27
1998	11	3	-	2	-	3	19
1999	10	4	3	6	1	-	24
2000	9	5	2	2	5	1	24
Total	57	17	6	14	6	16	Avg. 23.2

The future of the Graduate School

Teacher's enthusiasm for the Graduate School activity as well as qualified examinations for the degrees is clear indication of the necessity of the Graduate School. From the governmental point of view this kind of activity has been stated to be significant since the Ministry of Education stresses in the revised edition of *the Finnish Knowledge in Mathematics and Sciences in 2002* – program (12th of October, 1999) that the Graduate School for Mathematics, Physics and Chemistry Education will be continued and 1) the international co-operation in mathematics and science education research and development will be increased, 2) the equality in mathematics and science education research and development will be emphasised, and 3) the Academy of Finland proceeds to fund mathematics and science education research and development. However, in their recent decision the Academy of Finland did not grant any research positions for the Graduate School of Mathematics, Physics, and Chemistry Education. In the arguments it was shortly stated that the activity of the Graduate School is important but the scientific level does not stand comparison with the other mathematics, physics and chemistry graduate schools.

A Hidden Regulating Factor in Mathematics Classrooms: Mathematics-Related Beliefs

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This paper will focus on teachers' and pupils' mathematics-related beliefs which pertain to their subjective knowledge, and which act in mathematics classrooms as a hidden factor regulating the quality of mathematics teaching and learning. Some research-based examples of teachers' and pupils' beliefs are exposed and discussed. The meaning of beliefs for the teaching / learning process is dealt with. At the end, the possibilities to change those beliefs that are not optimal for the up-to-date understanding of learning and teaching are put forward, and some change solutions from the literature are discussed.

Introduction

Within school research, researchers have mainly focused on the cognitive results in the understanding of learning. However, affective by-results of learning that are in connection with an individual's meta-cognitions (Schoenfeld 1987), as a hidden factor, determine the quality of learning, though they are often put aside in research. During the last twenty years, mathematics educators all over the world have placed more and more emphases on the aspect of meta-cognition in learning, especially in the form of pupils' beliefs (Schoenfeld 1992). Beliefs are situated in the "twilight zone" between the cognitive and affective domain, since they have a component in both domains.

The purpose of this paper is to draw attention to the role and meaning that beliefs have in learning and teaching mathematics. Beliefs present some kind of tacit knowledge; everybody has his[1] personal tacit knowledge which is involved in every learning and teaching situation, but is rarely explicated. Beliefs differ from the scientific knowledge (objective knowledge) which can explicated and discussed. Teachers' and pupils' mathematics-related beliefs are dealt with mainly in the paper, but similar statements are also valid e.g. for university students and professors.

Theoretical background

An individual continuously receives perceptions from the world around him. According to his experiences and perceptions, he makes conclusions about different phenomena and their nature. The individual's personal knowledge, e.g. his beliefs, is a compound of these conclusions. Furthermore, he compares these beliefs with his new experiences and with the beliefs of other individuals, and thus his beliefs are under continuous evaluation and change. When he adopts a new belief, this will be automatically connected to a part of the larger structure of his personal knowledge, of his belief system, since beliefs never appear fully independently. Thus, the individual's belief system is a compound of his conscious and non-conscious beliefs, hypotheses or expectations and their combinations. (Green 1971)

Today we stress the active conception of learning behind which constructivism lies. According to a view of learning compatible with constructivism, a learner has to work actively in order to elaborate and to up-to-date his knowledge structure (e.g. Davis & al. 1990; Ahtee & Pehkonen 1994). The meaning of pupils' own beliefs about mathematics and its learning is emphasized as a regulating system of their knowledge structure. Since the teacher has a central role as an organizer of their learning environments, also his beliefs are essential for the success and quality of learning. Therefore, teachers' and pupils' mathematical beliefs play a key role when trying to understand their mathematical behavior (Noddings 1990).

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[1] If the gender is unknown, I have used the male form.

What are beliefs?

During the last ten years of belief research, some research synthesis have been published on mathematics-related beliefs (e.g. Underhill 1988; Thompson 1992; Pehkonen 1994; Op 't Eynde & al. 2001) which all try to clarify the fuzzy scenery. In the paper Furinghetti & Pehkonen (2001), the fuzziness of belief characterizations is discussed in detail, and they result the following recommendations. When dealing with beliefs and related terms, it is advisable

- to consider two types of knowledge (objective and subjective)
- to consider beliefs belonging to subjective knowledge
- to include affective factors in the belief systems, and distinguishing affective and cognitive beliefs, if needed
- to consider degrees of stability, and to acknowledge that beliefs are open to change
- to take care of the context (e.g. population, subject, etc) and the research goal in which beliefs are considered.

Therefore, it is of the uttermost importance to explain also here how one characterizes beliefs. We understand *beliefs* as an individual's subjective knowledge, which also includes his feelings, of a certain object or concern to which tenable grounds may not always be found in objective considerations. This definition is very near to the one given by Lester & al. (1989).

The individual himself set the reasons why a certain belief is adopted – usually non-consciously. The adoption of a belief may be based on some generally known facts (and beliefs) and on conclusions drawn from them. But each time, the individual makes his own choice of the facts (and beliefs) to be used as reasons, and his own evaluation on the acceptability of the belief in question. Thus, a belief, in comparison to objective knowledge, always contains an affective coloring (dimension). This dimension influences the role and the meaning of each belief in the individual's belief structure. (Green 1971)

Beliefs are usually held with a different degree of conviction (Abelson 1979). For example, Kaplan (1991) refers to the concepts "deep belief" and "surface belief", which could be understood as non-conscious beliefs and conscious beliefs. In the literature, authors often use conceptions as a parallel to beliefs. Here, we define *conceptions*, according to Saari (1983), as conscious beliefs, i.e. we understand conceptions[2] as a subset of beliefs. Thus, conceptions are higher order beliefs, they are based on such reasoning processes for which the premises are conscious. Therefore, there seems to be a reasoning basis for conceptions, at least they are justified and accepted by the individual himself.

An individual's mathematics-related beliefs are often divided into subgroups, e.g. beliefs about the nature of mathematics, beliefs about mathematics learning and teaching, and beliefs about oneself as a learner of mathematics (e.g. Underhill 1988). Such a division is actually artificial, in the sense that many beliefs belong to more than only one of these four groups. For example, pupils' beliefs about the nature of school mathematics influence their conceptions of how mathematics will be learned (and how it should be taught). Nevertheless, such a grouping of beliefs will help us to structure the situation.

Furthermore, an individual's beliefs are quasi-logically connected with each other, i.e. the individual himself defines the logic between the beliefs, and thus they form a belief system (Green 1971). Such a structure of the individual's mathematics-related beliefs is called his *view of mathematics*, which is a wide spectrum of his beliefs and conceptions. It contains, among others, four main components: (1) beliefs about mathematics, (2) beliefs about oneself as a learner and as a user of mathematics, (3) beliefs about mathematics teaching, and (4) beliefs about mathematics learning.

The question "What is mathematics?" goes back to the philosophy of mathematics. Mathematics philosophers (e.g. Ernest 1991; Hersh 1997) have stated that there are many different views of mathematics, of which the following three seem to be the most important one (see also Grigutsch & al. 1998): The instrumental view (mathematics is a "*tool-box*"), the Platonist view (mathematics is in the first place a *formal system*), and the constructivist view (mathematics is a *process* which is realized e.g. in problem solving).

Beliefs and knowledge

Often two parts are separated in knowledge: objective knowledge and subjective knowledge. *Objective knowledge* in mathematics means the generally accepted structure of mathematics, which is a compound of all mathematicians' research work for over more than 2000 years. This structure of mathematics is today so large that it is outside of any human being. The last mathematician who is said to possess an overview of the "whole mathematics" was Poincaré (e.g Boyer 1985, 650). When studying mathematics, we can only learn a part of it, and usually in our own characteristic way, i.e. we form our own conceptions on the topics to be learned. In the recent psychological research on learning, the concepts "learning" and "knowledge building" have been separated (e.g. Bereiter & Scardamalia 1996).

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[2] As for belief, there are many competing characterizations for "conception"

One of the main features of mathematical knowledge is its pure logic. A premise for objective knowledge is that all the beliefs that form its basis are logically justified and publicly accepted, in the sense that also all other facts in the world of phenomena speak for them. An individual's *subjective knowledge* is something unique which usually only he possesses, since it is based on his experiences and understandings.

There are many connections between an individual's subjective knowledge and objective knowledge. On one hand, an individual may study mathematics (objective knowledge), and thus enlarge his subjective knowledge. For example, the concept of function might be the topic of his study. But all the time, the knowledge he possesses on function belongs to his subjective knowledge, although his conception may asymptotically approach the official concept of function which concept pertain to objective knowledge (cf. the idea of knowledge building in Bereiter & Scardamalia 1996). Thus, his view of mathematics contains, among others, his conceptions of mathematics as a whole and in detail. On the other hand, an individual's subjective knowledge may enrich objective knowledge, when a certain piece of his subjective knowledge is publicly presented, justified, discussed, and socially accepted.

Sfard (1991, 3) goes deeper into the connection between beliefs and knowledge. She considers conceptions as the subjective /private side of the term 'concept' defined as follows: "*The word "concept" (sometimes replaced by "notion") will be mentioned whenever a mathematical idea is concerned in its "official" form as a theoretical construct within "the formal universe of ideal knowledge"*". Whereas she explains that "*the whole cluster of internal representations and associations evoked by the concept – the concept's counterpart in the internal, subjective "universe of human knowing" – will be referred to as "conception"*". The distinction between conception and knowledge is complicated by the fact that an individual's conception of a certain concept can be considered as a "picture" of that concept. Since a picture and its object are not the same, and usually the picture shows only one view on the object, similarly a conception represents only partly its object (concept).

Knowledge as a network

As mentioned, mathematics is a cultural artifact that is more than 2000 years old. Therefore, it can be considered as a human created network (structure) of knowledge. It is well-known that mathematical knowledge is divided into procedural knowledge, as algorithmic knowledge, and conceptual knowledge, as fact knowledge, which both are needed when learning mathematics (Hiebert & Lefevre 1986). Often knowledge structure is represented as a graph (e.g. Kiesswetter 1977) that is a compound of facts (knots) and their connections (paths). As a result of studying, which aims for understanding, new connections are constructed in the knowledge graph. In the best case, an individual's mathematical knowledge will be formed to be a clear logical network – a mathematical knowledge structure.

The meaning of a rich knowledge structure in mathematics is evident in problem solving situations. In the model of Kiesswetter (1983), a solution for a problem can be produced, if the solver is able to construct enough additional connections in his knowledge structure between the old facts. This procedure has near connections with creativity (e.g. Silver 1997).

But in practice most of each individual's mathematical knowledge is on belief level (cf. Sfard 1991). It is a compound of personal (more or less clear) conceptions or memory pictures on earlier learned mathematics that might also deviate significantly from the generally accepted conceptions. Furthermore, there are emotion-laden memories, mostly frustrations, in connection to an individual's knowledge structure (cf. with Bereiter's module theory of learning, Bereiter 1990).

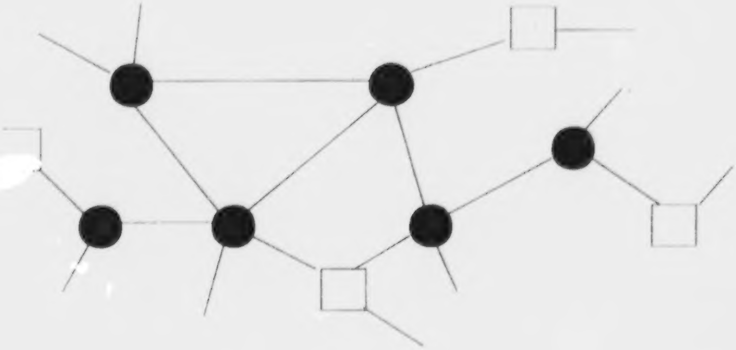

FIGURE 1. A schematic picture on an individual's knowledge structure as a graph (black circles represent facts and white squares emotions).

A mathematician strives toward an ideal, purely cognitive knowledge structure where knowledge units are only logically connected with each other without any emotional coloring. A well-known example of this was the bourbakist trial to structure the whole mathematics starting with the set theory (cf. Boyer 1985).

Beliefs in action

Especially in the early years of belief research, i.e. in the 1980's, several researchers tried to sketch what kind of beliefs teachers and pupils possess regarding mathematics and its learning and teaching (e.g. Frank 1988; Thompson 1989). Some researchers also have sketched beliefs conveyed by public opinion, i.e. in newspapers (e.g. Weber 1995). Further sources of classroom beliefs are derived, for example, through teaching materials and school administrational orders.

What kinds of beliefs may one observe in mathematics classes?

In mathematics lessons, teachers' and pupils' mathematics-related beliefs form an important influencing factor for the quality of teaching and learning. Additionally, pupils are under the influence of the beliefs of textbook authors, pupils' parents and relatives, etc. On the one hand, a teacher's beliefs conduct his pupils' learning. If the teacher considers mathematics primarily as calculations, then his pupils will have to calculate much during the mathematics lessons. On the other hand, pupils' beliefs direct their performance in learning situations. If pupils believe that mathematics is merely calculating and using ready-made formulas, they will have difficulties in problem solving, where one must first think and discover the kind of method to apply.

Many researchers have listed mathematics-related beliefs that pupils have revealed. Among others, Martha Frank (1988), based on her research, was able to extract the following five beliefs from pupils:

- Mathematics is computation.
- Mathematics problems should be quickly solvable in just a few steps.
- The goal of doing mathematics is to obtain "right answers".
- The role of the mathematics student is to receive mathematical knowledge and to demonstrate that it has been received.
- The role of the mathematics teacher is to transmit mathematical knowledge and to verify that students have received this knowledge.

Frank's study was conducted among a group of talented middle school pupils, but other researchers have obtained similar results from different pupil populations.

Also teachers' beliefs are investigated in many studies. For example, Alba Thompson (1989) condensed teachers' beliefs on problem solving, when interviewing some middle school mathematics teachers, into the following five statements:

- It is *the answer* that counts in mathematics, once one has an answer, the problem is done.
- One must get an answer in *the right* way.
- An answer to a mathematical question is usually a number.
- Every context (problem statement) is associated with a unique procedure for "getting" answers.
- The key to being successful in solving problems is knowing and remembering what to do.

Furthermore, she noticed that conventional mathematics teaching supports the development of such beliefs, both for pupils and teachers.

Both of these examples are from North-American researchers, but similar results have also been obtained in other countries, e.g. in Finland (cf. Pehkonen 1992, 1993). Furthermore, during teacher in-service courses, held by the author in Finland, in Estonia and in Germany, participants have confirmed that Frank's and Thompson's results also are valid in their countries. An interesting observation is that teachers' and pupils' beliefs about problem solving and related mathematics are very similar.

On the structure of belief systems

The structure of belief systems differs totally from that of a knowledge system. Within a knowledge system, one strives for the inner logic, since logic is one of the basic requirements for knowledge. Whereas in a belief system, an individual tries to be logical, though usually the result is somewhat quasi-logical, i.e. he puts himself forward the rules of his logic and his axioms. Based on these kinds of observations, Green (1971) has introduced three dimensions of belief systems: quasi-logicalness, psychological centrality and cluster structure (see more e.g. Pehkonen 1994). As a fourth dimension, the degree of conviction that is due to Abelson (1979) is discussed here.

Level of conviction

Kaplan (1991) introduced two levels of beliefs: deep beliefs and surface beliefs, but this bipolar division is too rough. We need more levels than these two, in order to keep our statements powerful. As a solution we might accept a wide spectrum of beliefs: deep beliefs are at one extreme, and surface beliefs at the opposite. Let us consider as an example a researcher's beliefs. If he has learned to use only statistical methods in his research, the positivist paradigm is for him self-evident. It is not a question to discuss about, because he *knows*, and therefore, does not accept any other method as research. Thus, beliefs connected with this paradigm situation are very deep-rooted. Whereas, for example, the following one may be a surface belief: "*In the list of references, one should always use the* APA *guidelines*". Between these beliefs, there are many levels of certainty for beliefs.

Such deep-rooted beliefs (e.g. those connected with research paradigm) are self-evident to the individual in question, and therefore, their certainty for him is 100%. They form the "axioms" in the individual's worldview. Nevertheless, the individual might possess beliefs of which he is 100 % sure, but which he is ready to change when there is enough evidence for that. For example, the belief "*Seville is situated in the middle of Spain*" could be changed easily, since a look at a reliable map shows that Seville is almost on the seaside. The difference between these two types of beliefs held with a 100 % certainty might lie in the amount of affect in them and in their psychological centrality.

Most beliefs are held with a lower amount of certainty than 100 %. For example, a pupil may give the following characterization: "*In order that an algebraic expression is an equation, there should exist x in some form*", but if you ask how sure he is of his answer, he may say e.g. "*about 80 % sure*". One of the topics in the dissertation of Kaarina Merenluoto (2001) are such certainty estimations of pupils on their responses.

The meaning of beliefs for mathematics teaching and learning

The central role of beliefs for the successful learning of mathematics has been pointed out again and again by several mathematics educators (cf. Schoenfeld 1992). In this connection, the following points are given as an explanation for these effects: Beliefs may have a powerful impact on how children learn and use mathematics, and therefore, beliefs may also form an obstacle for the effective learning of mathematics. Pupils who have rigid and negative beliefs of mathematics and its learning easily become passive learners, who emphasize remembering more than understanding in learning.

Beliefs and learning form a devil's circle (Spangler 1992): On the one hand, pupils' experiences in mathematics learning influence and form their beliefs. On the other hand, beliefs have an effect on how pupils will behave in mathematical learning situations, and thus conduct their ability to learn mathematics. Therefore, pupils' beliefs, revealed through research, reflect teaching practices in the classroom.

Mathematical beliefs as a regulating system

In her dissertation, Martha Frank (1985) introduced a schematic picture of some factors affecting pupils' problem-solving behavior. Since most of the factors act via pupils' belief systems (their view of mathematics), I have organized the components in the scheme in another way (Figure 2). This scheme, in fact, emphasizes the regulating character of a pupil's view of mathematics.

Beliefs play a central role as a background factor for pupils' thinking and acting. A pupil's mathematical beliefs act as a filter that regulates almost all his thoughts and actions concerning mathematics. A pupil's prior experiences of mathematics affect fully at the level of his beliefs – usually non-consciously. When he uses his mathematical knowledge, his beliefs are also highly involved.

In contrast to this, a pupil's motivation and needs as a learner of mathematics are not always connected with his mathematical beliefs. Additionally, there are many societal mathematical myths, e.g. that mathematics is merely calculation (for more myths see e.g. Frank 1990 or Paulos 1992); these myths also influence a pupil's mathematical behavior via his belief system.

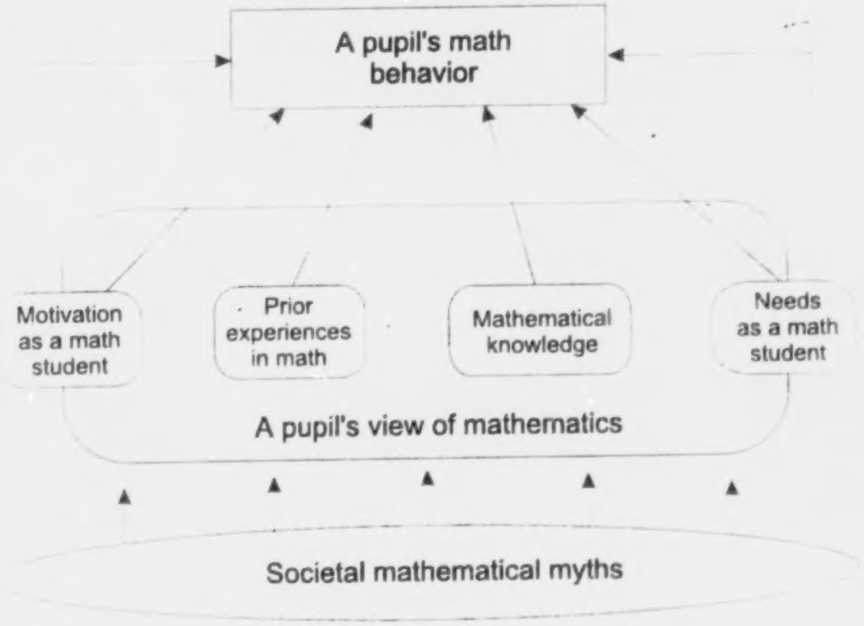
A pupil's math behavior
Motivation as a math student
Prior experiences in math
Mathematical knowledge
Needs as a math student
A pupil's view of mathematics
Societal mathematical myths

FIGURE 2. Factors affecting pupils' mathematical behavior.

The scheme of Figure 2 shows such a situation in which a pupil's mathematical performance is influenced by several factors that affect through a system or a net of his own beliefs. However, this is only part of the truth, in fact, the situation is much more complex. Pupils act within a very complex net of influences – Underhill (1990) speaks about a web of beliefs. For example, their mathematics teacher, classmates, friends, parents, relatives and teachers of other subjects all have their own views of mathematics and its teaching and learning. These beliefs more or less affect learners' beliefs, and usually in a contradicting way. Furinghetti (1997) also pointed out in a recent paper a similar complexity in the influence of beliefs.

Mathematical beliefs as an indicator

There is another practical meaning of beliefs: An individual's view of mathematics (mathematical beliefs) may form a practical indicator in a situation which one is not otherwise able to observe [3]. Since the view of mathematics transmitted through beliefs, expressed by an individual, gives a good estimation of his experiences within mathematics learning and teaching, we have a method to indirectly evaluate the instruction he has received/has given: In the case of a teacher, the view of mathematics may act as an indicator

(1) of teachers' university studies,
(2) of teachers' professional views,
(3) of teachers' in-service training.

In the case of pupils and students, the view of mathematics could function as an indicator

(4) of students' experienced teaching (in schools and universities).

Generally, one may consider the view of mathematics as an indicator

(5) of the functioning of the whole school system.

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[3] The idea of beliefs as an indicator of implemented instruction can be attributed to Günter Törner (University of Duisburg).

Mathematics teaching forms a part of the general education provided by the school that is implemented within a societal context. In the research done (e.g. Pehkonen & Törner 1999), one may see connections with the change processes within the society, connections arising outside the framework of mathematics education. Thus, the view of mathematics also has a role as an indicator

(6) of social sensitivity.

Mathematical beliefs as an inertia force

If we aim to develop mathematics teaching in schools, we are compelled to take into account teachers' beliefs (their view of mathematics), and also pupils' beliefs. Usually, the question is of experienced teachers' rigid attitudes and their steady teaching styles, which will act as an inertia force for change. Thus, the problem is how to help teachers to develop and enlarge their own pedagogical knowledge. Also, pupils' view of mathematics might not be optimal for the up-to-date understanding of learning and teaching, and therefore, the conditions of change should be considered. Thus, beliefs have a central position when trying to change teaching.

Changing beliefs

Since teacher change (i.e. a teacher's professional growth) is still a "hot" topic in mathematics education, there are many interesting longitudinal research projects reported during the last years (e.g. Fennema & al. 1996, Cooney & al. 1998, Chapman 1999). In the literature, there are also several descriptions of trials to change pupils' beliefs (Franke & al. 1997b; Wood & al. 1997), as well as edited books dealing with the change problems (e.g. Fennema & Nelson 1997).

Conditions for teacher change

To change an individual's beliefs is a long-term process, and this usually demands the cooperation of the object person. It is not possible to force somebody to change his beliefs. Such a training that aims to involuntarily change one's beliefs is called indoctrination – the common name "brainwash" is a very fitting description for it. Therefore, speaking about changing beliefs should be understood as "offering opportunities for change".

If someone from outside the school aims to achieve a change, he should keep an eye on several factors. The experiences revealed that, for example, the support given to teachers in their schools was essential for teacher change. It was important to visit other classrooms and discuss observations made with colleagues, in order to achieve a change of beliefs on the practical level. It also seemed to be important to organize such situations in which teachers could reflect on their thinking and actions.

One of the basic requirements for change seems to be perturbance. An individual should see himself that something does not match in his own belief system. Based on research experiences, the following conditions of teacher change could be explicated (Shaw & al. 1991): In order to affect a successful and positive change, (1) teachers need to be perturbed in their thinking and actions, and (2) they need to commit to do something about the perturbance. In addition, (3) they should have a vision of what they would like to see in their classrooms, and (4) develop a plan to realize their vision (Figure 3).

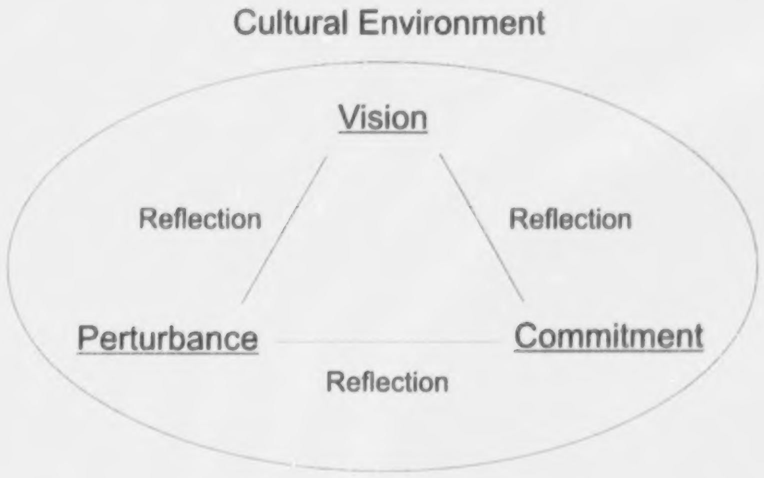

FIGURE 3. Framework for teacher change (Shaw & al. 1991).

In the framework (Figure 3), there are four central concepts. The cultural environment is different for each teacher. For each teacher in the research project of Shaw & al. (1991), some central cultural elements were noticed which have their impact on the process of change. Such elements are e.g. the support given by others, time, money, other resources, taboos, customs, and common beliefs. The change cannot happen without a perturbance in a teacher's thinking and actions. For example, pupils, colleagues, parents, administrators, teacher-educators, books, articles, and self-reflection may act as sources of perturbance[4]. Commitment is a personal decision to implement the change as a result of one or more perturbances. For teachers in the process of change, they need to form a personal vision of what mathematics teaching and learning should look like in their classes. Thus, if we want to have a change in teaching, teachers should already be actively involved in the planning stage of the innovation.

Edwards (1994) exposed a similar model. He stated that there are six factors which "drive the teacher change process: (1) experiencing a perturbation, (2) having commitment to change, (3) constructing a vision what specific changes might look like within a teacher's own classroom, (4) projecting the teacher's self into that vision, (5) deciding to make changes within a given context, and (6) being a reflective practitioner". Thus he developed further the model of Shaw & al. (1991) by adding three more factors to the originally three (perturbance, commitment, vision).

The component of having a personal vision seems to be a powerful, but often neglected factor in the formation and development of beliefs. For example, Presmeg (1993) has conducted research on it in a larger context of imagery. She emphasizes that when trying to change teachers' visions, one must get on the level of teachers' imagery, since imagery imparts an "affective coloring" to memory.

As an example of a practical situation, teacher change described by Cobb & al. (1990) fits in very well with the explained theory. The teacher in question was willing to cooperate and planned with the researchers new ways to teach. But the researchers noticed that, in classroom situations as soon as the researchers were no more there, she used her old conceptions of mathematics teaching when making her decisions. The change occurred until when her thinking became perturbed through observing with interviews that the teaching strategies she used were not powerful enough. In the book of Fennema & Nelson (1997), there are described several research project aiming to teacher change.

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[4] Pehkonen & Törner (1999) revealed a similar list of change factors in interviews with German teachers.

Development of teachers' beliefs

If a teacher is willing to change his own beliefs, the first phase is to be aware of and reflect on them. Here the following theoretical model of teacher change, exposed by Thompson (1991), might be helpful. It sketches some levels in the development of teachers' conceptions of mathematics teaching. Thompson proposed a framework for the development, and this is based on her own reflections of the training carried out with twelve pre-service and in-service teachers over five years. "*The proposed framework consists of three levels … Each level is characterized by conceptions of:*

1. *What mathematics is.*
2. *What it means to learn mathematics.*
3. *What one teaches when teaching mathematics.*
4. *What the roles of the teacher and the students should be.*
5. *What constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions.*"

Her model is partly similar to the earlier published considerations of Schram & Wilcox (1988) and Schram & al. (1989).

In her paper, Thompson (1991) gave a brief verbal characterization of each level. Based on these, I have elaborated a table of the characterizations (Table 4), in order to give a better overview. I reduced the five components of the Thompson model to four by combining the second and third components (learning and teaching mathematics). In addition, I added the development of conceptions in problem solving as a fifth column.

In the first column, the conception of mathematics is developed from rote calculations to the complex system of interconnected mathematical entities. In the second column, the conception of learning and teaching mathematics is changed from mere memorization to understanding via doing mathematics.

In the third column, the role of the teacher develops from a demonstrator to a facilitator of learning, and the role of pupils from imitators to active learners. In the fourth column, criteria for judging correctness are changed from the teacher's authority to self-regulated action. In the fifth column, the conception of problem solving is developed from solving separate "story problems" to a teaching method.

TABLE 4. A modified set of the Thompson levels in the development of teachers' conceptions of mathematics teaching.

	What is mathematics?	**What is learning /teaching mathematics?**	**What are the roles of the teacher and pupils?**	**What are the criteria for judging correctness?**	**What is problem solving?**
LEVEL 0	• common uses of arithmetic skills in daily situations • mathematical knowledge means rote, procedural proficiency	• memorization of collections of facts, rules, formulas and procedures • teaching sequences of topics and skills specified in a book	• the teacher is a demonstrator of well-established procedures • pupils imitate	• the teacher is an authority for correctness • accurate answers as the goal of mathematics instruction	• getting answers to "story problems" • helping pupils to identify the right procedure ("rules of thumb")
LEVEL 1	• rules continue to govern all work in mathematics • appreciation for understanding the concepts and principles behind rules	• an emerging awareness of the use of instructional representations • use of manipulatives in instruction • promote the view that "math is fun"	• much as in Level 0 • the teacher attends to the "reasons behind the rules" • pupils include some understanding	• authority for correctness still lies with experts	• viewed as a separate curricular strand • taught separately • problems unrelated to mathematical topics being studied • teaching "about" problem solving
LEVEL 2	• understanding mathematics as a complex system of different interconnected concepts, procedures and representations	• teach for understanding • understanding grows out of engagement in the process of doing mathematics	• the teacher steers pupils' thinking in mathematically productive ways • the teacher listens to pupils' ideas • pupils express their ideas	• the process of doing mathematics is the goal of teaching • pupils themselves check their answers for correctness	• problem solving is used as a teaching method • teaching "via" problem solving

Thompson (1991) observed three levels in teachers' development (Table 4). But it is worthwhile noting that actually the researcher's beliefs will determine the framework, within which the development of teachers' conceptions of teaching mathematics is considered. One may imagine that, in the future, some teachers will develop themselves further, e.g. into the third or the fourth level (level 3 and 4), whatever they are.

Thompson's model has been used e.g. by Sinikka Lindgren in evaluation of the levels of teacher students' mathematical beliefs (e.g. Lindgren 1996). Within the research project "Developments in Pupils' Beliefs" (Hannula & al. 1996), we tried to create an indicator for the levels of pupils' beliefs that is based on Thompson's model. The fitting of the model was checked through investigating both of its extreme poles: On the one hand, the conceptions of school beginners about mathematics were clarified (cf. Pehkonen 1995b) – they should be on the most elementary level (level 0) or on the pre-zero level. And on the other hand, the most developed mathematical conceptions (level 2) were investigated. Here one possible group of test subjects is formed by professional mathematicians from institutes of mathematics (cf. Pehkonen 1999).

Also Franke & al. (1997a, 264) have developed levels of teacher change "*that encompass information about a teacher's beliefs, knowledge, and use of children's thinking in classroom practice*". They used four general levels with the fourth level being a compound of two sublevels. The levels form a hierarchic structure, each building on the previous one.

Is it possible to change pupils' beliefs?

If a teacher wants to change some of his pupils' beliefs, such as "*The textbook should be worked through page by page, and all the tasks should be done*", he should be prepared for a long process. About ten years ago I acted as a teacher in a grade 7 class beside of my activities as a teacher educator. When beginning with a new class, I did not introduce a textbook for a couple of weeks, since I realized that the pupils needed repetition, and I became acquainted with my new pupils' skills by using some proper problem-centered tasks. As the pupils then after two weeks said that they should also have a textbook, since parallel classes had already been calculating from the textbook a long time, I promised to give them the books immediately in the next lesson. Then we calculated from the textbook diligently, and I copied the answer sheets for the pupils' use, in order to let them check the tasks independently. During those lessons, the pupils only calculated and checked their answers, nothing else. After about a week, they began to ask whether we would be doing something else besides calculating only from the textbook. From then on, I had no difficulties with the pupils when skipping around in the textbook, as long as I explained why to them.

Optimal mathematics teaching for developing proper beliefs

Shulman (1987) has stressed that a teacher needs beside his content knowledge also pedagogical knowledge that he labels to pedagogical content knowledge. As a hypothesis one may say that instructional circumstances for developing optimal pupils' beliefs require from the teacher *content knowledge* deep enough, proper *pedagogical content knowledge*, the *most developed view of mathematics* as well as *flexibility* in implementing instruction. None of these four aspects can be left aside. The first three requirements are clear for a teacher's ability to act as a mathematics teacher. But in addition, the teacher should be flexible to take into account pupils' needs and earlier beliefs in planning instruction and in implementing it.

One important connected aspect in a teacher's flexibility is the sharing of authority in instruction, i.e. how much freedom and responsibility the teacher gives his pupils for their learning. The sharing of authority has been one of the central themes in international discussions on improving mathematics teaching (cf. Cooney 1993). If a pupil is compelled to take responsibility for his own learning, his view of learning and mathematics will change (e.g. Hart 1993). The other side of the coin is pupils' autonomy in learning mathematics that is discussed e.g. in the published paper of Furinghetti & Pehkonen (2000).

From these aspects, one can deduce some criteria for teaching: From a pupil's view point, this change of focus is connected essentially with his more personal attitude toward mathematics and its learning (cf. Hoskonen 1999). The mathematics to be learnt then has a *meaning* for the pupil. Secondly, the striving to *understand* the topics and their relationships is connected organically, if possible, with the teaching and learning, compatible with the developed view of mathematics. The third important factor is the promoting of *application of mathematics* and *skills to use* mathematics. One additional factor that supports effective learning is mathematical *communication* between pupils; the communication can be fostered i.a. with different forms of cooperative learning (cf. NCTM 1996).

One possible method to help a teacher to create such a learning environment advantageous to pupils is the overall accepted "open approach" which was developed in Japan in the 1970s (e.g. Nohda 1991, 2000). For this, one can use open-ended tasks that have proven to be a promising solution for generating a proper learning environment; for examples see Pehkonen (1997). They seem to offer an opportunity for such teaching and learning that is more meaningful from pupils' viewpoint. The influence of open-ended tasks on pupils' motivation has also been clarified in Finland (cf. Pehkonen 1995a). The main result in the study was that pupils were clearly more interested in these types of tasks, but that the teacher and his way to realize teaching was the most influential factor.

Conclusion

With the rise of constructivism, teachers' and pupils' beliefs are seen as a paramount factor for understanding their performance in teaching and learning. An individual's beliefs act as invisible glasses through which he observes the world around. These glasses "color" his perceptions and thus further his understanding of the world around and its phenomena. In that sense, the individual's beliefs form a filter that regulates his performance in mathematics.

When aiming a change in school teaching, one should remember that constructivism stresses the meaning of beliefs as a "real-life" factor that should then be counted. Beliefs might form a real obstacle for change or, at least, act as a strong inertia force against the change. This means that the change will not happen in short time period. The length of the change depends on the depth of beliefs that should be changed (cf. Kaplan 1991).

Also in the case of educating new teachers, we should be aware of the meaning of beliefs. If we want to change mathematics teaching in schools, one possible way is to try to reach a change in teacher pre-service education. And this means that all three components of teacher education (departments of mathematics, departments of teacher education, practice schools) should convoy similar messages to students. Students' beliefs are more formed by such hidden messages (e.g. how learning happens) than direct lecturing. Already in the beginning of the 1960's, Marshall McLuhan said that "*the medium is the message*".

Therefore, all teacher educators – both in departments of mathematics and departments of teacher education – are in key positions, regarding the development of teacher students' mathematical beliefs. We know that conventional mathematics teaching creates certain types of beliefs, and the resulting view of mathematics is considered as a too restricted one. On the one hand, also in departments of mathematics, professors should consciously strive for enlarging a student's view of mathematics by using different types of learning methods. On the other hand, teacher educators in departments of teacher education and teachers in practice schools are key elements in changing the dominant teaching tradition: If their own teaching does not contain enough components, generating new ideas which are also implemented (cf. Pehkonen 1998), prospective teachers will go to schools using their old conceptions and practices. It is not enough to talk about the theory of change, one should reflect and model it!

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Primary School Teachers' Mathematics Beliefs, Teaching Practices and Use of Textbooks

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The study-in-progress underlying this paper investigates relationships between 7- to 8-year-old children's teachers' beliefs and their mathematics teaching practices – especially the use of textbooks. Data were gathered through a belief questionnaire from 140 teachers, videotaped classroom observations and interviews from 6 teachers. Analyse includes the categorisation and comparison of beliefs and practice, particularly the relationship between beliefs and the use of mathematics textbooks. Findings indicate that there were inconsistencies between beliefs and teaching practices. Teachers' recollections of mathematics studies at school and their dependence on mathematics textbooks influenced their teaching practices.

Background

The role of the mathematics' textbook is essential when the standard mathematics lesson begins with some initial teacher exposition, and is followed by the children working through the same exercises in their textbooks. The quickest children count some extra exercises from their textbooks. The homework is also a further exercise from the textbook. (Kallonen-Rönkkö 1997, 264.) The right answers are found from answer keys in the classroom, and no attention is paid to the different solution models of exercises. The right answer is more important than the different solutions. This kind of traditional mathematics teaching model is still alive in the mathematics' textbooks. My licentiate study gives support to this fact. In my licentiate study (1998) I analysed both the exercise and the concept structures of the two series of first and the second grade mathematics textbooks published before and after the Finnish curriculum reform in 1994. The study showed that the structure of exercises in mathematics textbooks supports traditional mathematics teaching in which most of the lesson is quiet counting. According to Lerman (1993, 71), this kind of traditional teaching and teachers' dependence on the textbooks gives the text, the textbook and the textbook writers complete authority in the classroom. Furthermore, the structure of first and the second grade mathematics textbooks does not reveal the whole picture. According to Pehkonen (1998), a teacher's system of beliefs guides his practices and also filters and interprets the teacher's perceptions. To capture the whole picture about mathematics teaching at the first and second grade in primary school, I decided to study the relationships between teachers' mathematics beliefs, mathematics teaching practices, and the dependence on mathematics textbooks. In my study the individual's beliefs are understood to be composed of his subjective experience-based implicit knowledge of mathematics and its teaching and learning. According to Pehkonen (1998, 45), "conceptions are understood to be conscious beliefs. The beliefs – conscious and unconscious – can be seen as a belief system." When the object of the belief system is mathematics or mathematics teaching/learning I use the term: "view of mathematics".

The view of mathematics

Ernest (1989, 250) has distinguished three different conceptions of the nature of mathematics, which answer the question "What is mathematics?". These are "the instrumentalist view, the Platonist view, and the problem solving view". The teacher's view of mathematics is the basis for the teacher's mental models of teaching and learning mathematics. When the teacher has an instrumentalist view of mathematics, a typical feature of his teaching is the strict following of teaching models and, for example, the strict following of a textbook's text and scheme. The teacher is a distributor of knowledge and completed rules. Typical of this model is children's compliant behaviour and mastery of skills model. The children are counting and, utilising rules and procedures. In the Platonic view of mathematics the role of the teacher can be seen as an explainer. This model is likely to be associated with conceptual understanding with unified knowledge. Children's learning is still the reception of knowledge model. Thus mathematics is discovered, not created. In the problem-solving view of mathematics the teacher is a facilitator and a child is an active constructor of knowledge. This model is based on the child's active construction of understanding model and exploration and autonomous pursuit of own interest's model. (Ernest 1989, 251; see also Dionne 1984, 223–224.)

According to Pehkonen (1998), beliefs can have a strong effect on how children learn mathematics. Children's learning experiences affect their beliefs, and on the other hand their beliefs influence how children approach new mathematical learning situations. Kaasila's study (2000) gives hints "how the recollections of teachers have a central significance for the images pupils have of mathematics itself, of the teaching of mathematics, of the role of a student in a class". Also according to Grigutsch (1998, 185), pupils' view of mathematics is influenced by the teacher as a person and his practices in mathematics education.

Method

I make use of quantitative and qualitative data in my study. A Likert-scale belief questionnaire was sent to first and the second grade teachers (N=230) in primary schools. The belief questionnaire included 70 statements concerning teachers' beliefs and conceptions about mathematics, learning mathematics, teaching mathematics, and mathematics teaching practices. To make comparisons, the teachers' answers were classified on the following scale: traditional, primarily traditional, mixed, primarily non-traditional, and non-traditional. For the basis of this classification there were the previously mentioned three Ernest's views of mathematics. The traditional answers were near the instrumentalist view of mathematics, and the non-traditional answers were near the problem solving view of mathematics.

On the basis of the questionnaire I selected 6 representative the first and the second grades' teachers from different primary schools. Teaching experience of these 6 teachers was between two to almost thirty years. Each teacher was contacted first by phone and all of them volunteered to participate in the study. Data collection for each teacher included: (a) two week's classroom observations (video-taped), (b) teachers' written lesson plans, (c) after the classroom observation period an interview (the questions during the interview focused on the nature of mathematics, mathematics teaching and learning practices and mathematics textbooks, assessment, school recollections about mathematics and teachers' mathematics studies) (d) analysis of the video-taped mathematics lessons together with the teacher.

Results

Most of the teachers (N=140) answering the questionnaire had beliefs that were between primarily traditional and primarily non-traditional. The beliefs of those who had only a few years teaching experience were nearly non-traditional (33.9 %). Kupari (1999) has obtained the same kind of results.

Notes from classroom observations and interviews from those 6 teachers are used to describe consistencies and inconsistencies between teachers' beliefs and teaching practice. According to Ernest (1989, 252–253), there are three key elements that influence the practice of mathematics teaching: "the powerful influence of the social context; the teacher's level of consciousness of his own beliefs and the extent to which the teacher reflects on his practice of mathematics." I found that teachers' recollections of their experiences in mathematics at school influenced their teaching practices. Teachers' school-time recollections, e.g. difficulties in mathematics learning, their school-time teacher's dependence on mathematics textbooks and inclination to the rules and routines methods, have significance for their teaching practice. The study of Lindgren (1998) gives support to this. In Lindgren's study (1998, 344) "special attention was paid to the emotional memories of the students' math teachers". Her interviews with student teachers revealed hints for "correlation between recalled math teachers' harshness, sharpness, and dependence on mathematics schoolbooks with student teacher's distrust of the open-approach method and inclination to the rules and routines method."

The use of manipulative in the teaching/learning situations was often regarded as useful for promoting the view that "math is fun". For example teacher's beliefs about mathematics were primarily non-traditional but his instructional practice was still focused on textbooks, rules, and procedures. There appeared inconsistencies between their beliefs and practices. Thompson (1984), Brown (1985) and Cooney (1985) have identified inconsistencies between the relationships of beliefs and classroom actions, too. They consider this relationship complicated because many social norms with possibilities and limits are connected: the values, the expectations and the beliefs of pupils, parents, and administrators. The influence of the curriculum, the practices of assessment and the dominating views and values of learning are connected. Those factors mentioned above have an effect on teacher's instruction.

Sometimes teacher's school-time recollections, e.g. difficulties in mathematics learning, were so deep that he wanted to protect the children – he did not give children space to think. Interviews and observations revealed that teachers should have more deepened mathematics studies during teacher education. Because of the uncertainty of mathematics, teachers mainly followed the order and the instructions of the mathematics books. The right answers were more important than solution procedures. Next I restrict my attention to case of Paula.

The case of Paula

Paula's teaching philosophy:

> *"I think it is this understanding mathematics by doing it through activities and also co-operative learning. I think that if you put such children who have difficulties in mathematics together with those who are good in mathematics they can help each other. Those who know story problems can teach those who do not know story problems."*

Paula has been a teacher over twenty years. She taught the first class. According to the questionnaire, Paula's answers about mathematics were rules and strict orders but also problem solving and creativity. On the basis of Paula's answers, mathematics learning meant for Paula children's creativity, concrete learning environments and also confidence in the power of mathematics textbooks and their complete patterns as though those patterns would have formed some kind of limits for her on how to learn mathematics. Paula's answers about mathematics teaching were emphasising understanding through problem solving and the use of manipulative, children's own solutions but also mathematics exactness and rote learning and routines.

During the interview Paula explained that she had had difficulties in mathematics at school as a child:

> *"I like to teach mathematics. Very often I am recollecting how I had difficulties in story problems and mental arithmetics in mathematics in the third and the fourth grade in primary school. I remember that I was very poor in mathematics. That is why mathematics teaching is to me a challenge and it is very important for me to do mathematics teaching practice meaningful to these children."*

Especially the story problems had been difficult for her and she wanted that her pupils should learn mathematics properly.

During the classroom observations I found that the mathematics textbooks were sometimes very dominating. For example Paula was so dependent on the textbooks that she did not hear child's right explanation of the right solution or she did not give any space to children's thinking – she only waited right answers or sentences that should be filled in the textbooks. After the classroom observation Paula analysed her teaching and her dependence on the mathematics the textbooks like this:

> *"Here my teaching is teacher-directed because I thought that you should see this how I teach this addition over ten. If I had more time I could have used manipulative and co-operative working....because the children can't read so well at the first grade we have to do these story problems together. It is also important to give to children good experiences in mathematics and by doing together story problems everyone can calculate them. There are still those who can do these problems alone."*

Comments: Although the children worked sometimes together and had activities, I found that the mathematics textbooks were very dominating. All her lessons were not teacher directed but textbook-centered. Because of her own difficulties in mathematics during her school-time she wanted her pupils to learn mathematics properly and should not have the same kind of difficulties in mathematics.

Conclusion

In teacher education we should pay more attention to students' own thinking and reflection. According to Raymond (1997, 574), "early and continued reflection about mathematics beliefs and practices, beginning in teacher preparation, may be the key to improving the quality of mathematics instruction and minimising inconsistencies between beliefs and practice." Also teachers should not work in such isolation but as members of learning communities. It is important to encourage them to work and pursue new ideas together because by considering both the positive and negative consequences of various teaching practices teachers would come to a better understanding their own beliefs and would consider whether they are consistent with their goals for their pupils (Barnett 1998, 92). In teacher preparation it is important to pay more attention to learning theories and their connection to the mathematics teaching and learning situations. We should read the child and know how she or he learns. It is not the most important thing to cover the textbooks and give right answers. If we do not listen to children's answers the children become uncertain and they will eventually answer what they think the teacher expects.

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Mathematics for Primary School Teachers

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In this paper I shortly describe those things I have learnt while trying to find out what mathematics prospective primary school teachers should learn during their teacher education. Main result is that as important as it is to think **what** to teach, is to think **how** to teach and to **whom** we teach. The ways of teaching and learning should be the same what we expect our students to use whith their pupils in future. And we should be more conscious of our students beliefs concerning mathematics and its teaching and learning.

Introduction

In Finland primary school teachers work in grades 1–6 where pupils are from 7 to 12 years old. Teachers are "class teachers" and thus they have to be able to teach every subject and, for example, primary school teachers teach over 70 % of the mathematics, which is taught to children during the period of compulsory education.

Future teachers pursue their studies at Universities, in Departments of Teacher Education, and their studies include a minimum of 160 credit units (one credit unit = 40 hours of work). Mathematics accounts for 3–4 credits of compulsory studies. Students then have a possibility to acquire 15 credits of mathematics as optional studies, and the Department of Mathematics offers these studies.

At the university of Joensuu optional mathematics courses have been very popular during last seven years. Mostly because they are held during summertime and weekends, and because teachers get more salary when they have done this mathematics unit, named mathematics approbatur (15 credits). The problem is that students (and teachers) feel that those studies are not useful when teaching – they give so little help for real classroom situations.

In my study I have tried to find out if there exists such mathematics which develops teachers skills in teaching mathematics, which gives teachers not only mathematics substance knowledge but also that kind of mathematical know-how which makes persons to be better teachers of mathematics?

Theoretical background

This study belongs to the field of belief research. It starts from the assumption that beliefs are related on practices (Pajares 1992; Thompson 1992; Lindgren 1995 and 1997; Pehkonen 1998 and Kupari 1999) and the following model by Ernest has been used as framework (Figure 1).

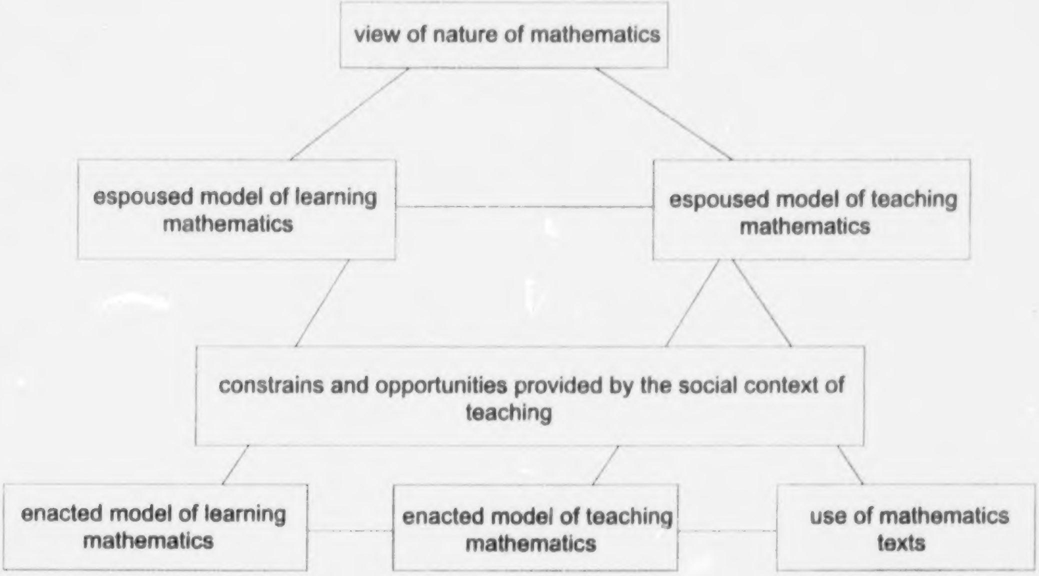

view of nature of mathematics
espoused model of learning mathematics
espoused model of teaching mathematics
constrains and opportunities provided by the social context of teaching
enacted model of learning mathematics
enacted model of teaching mathematics
use of mathematics texts

FIGURE 1. Relationship between Beliefs, and their Impact on Practice (Ernest 1989a, 252)

In Ernest's model, a view of the nature of mathematics provides a basis for teachers' mental models of the teaching and learning of mathematics, and although Ernest is well aware that social context can disturb this system, the main idea is that practices can develop only with the development of the individual's view of the nature of mathematics.

Since teachers' mathematical views consist of mathematical beliefs, we next concentrate on teachers' beliefs of mathematics and its teaching and learning. Beliefs are here understood as one's stable subjective knowledge (which also includes his feelings) of a certain object or concern to which tenable grounds may not always be found in objective considerations, and conceptions are conscious beliefs (Pehkonen 1995).

Although there exist several classifications applicable to teachers' mathematical beliefs (e.g. Dionne 1984; Ernest 1989b; Grigutsch et al. 1995; and Thompson 1992), the classifications seem to be rather analogous and the beliefs can be divided into three main categories. Figure 2 sums up and divides beliefs into three views of mathematics and its teaching and learning.

	beliefs concerning the nature of mathematics	**beliefs concerning teaching and learning of mathematics**
I	traditional/instrumentalist/toolbox aspect	teacher as instructor, mastery of skills with correct performance, content oriented with emphasis on instruction Thompson level 0
II	formalist/Platonist/system aspect	teacher as explainer, conceptual teacher as explainer, conceptual knowledge, content oriented with emphasis on conceptual understanding Thompson level 1
III	constructivist/constructivist/process aspect	teacher as facilitator, confident problem posing and problem solving, pupil oriented Thompson level 2

FIGURE 2. Compilation of teachers' beliefs concerning school mathematics and its learning and teaching

Because the descriptions of Thompson's levels 0, 1, 2 simultaneously portray, both the conception of the nature of mathematics and the conception of teaching and learning it is taken here to serve as a representative of levels I, II, III.

I At **Level 0** the conception of mathematics is based on perceptions of common uses of arithmetic skills. This implies that instructional practice focuses on facts, rules, formulas, and procedures. The role of the teacher is perceived as a demonstrator of well-established procedures, and the pupils' role is to imitate the demonstrated procedures and to practice them diligently. Obtaining accurate answers is viewed as the goal of mathematics instruction. The assessment of the correctness or accuracy of the answers is the responsibility of the teacher. Problem solving is viewed as getting right answers – usually by using prescribed techniques – to 'story problems'.

II At **Level 1** the conception of mathematics is broadened to include an emerging valuation for understanding the concepts and principles 'behind the rules'. However rules still play a basic part of mathematics. The teaching of mathematics is characterised by an awareness of the use of instructional representations and manipulatives. Teaching for conceptual understanding is not viewed as a central goal. Thus the use of manipulatives is often regarded as useful for promoting the view that 'maths is fun'. The connections between the actions performed on objects, the verbalisation of those actions, and their representations in mathematical symbolism is not explicitly discussed. The role of the teacher is perceived somewhat as in Level 0. Views of the role of the pupils are broadened to include some understanding. Problem solving is regarded as important in the mathematics curriculum but it is viewed as a separate strand from the 'traditional content'.

III At **Level 2** the conception of how mathematics should be taught is characterised by a view that pupils themselves must engage in mathematical investigation. The view of teaching for understanding that begins to develop at Level 1 is transposed to a view that understanding grows out of engagement in the very processes of doing mathematics. The physical and pictorial representations are regarded as a provision both for doing tasks carefully designed by the teacher and for generating non-standard procedures by the pupils. The role of the teacher is to steer pupils' thinking in mathematically productive ways. The questions intend to stimulate, guide, and focus pupils' thinking. The pupils are given opportunities to express their ideas and the teacher listens to and assesses their reasoning. (Thompson 1991)

When we think about what kind of experiences of school mathematics our students have, we can probably conclude that, based on their own mathematical background, their beliefs settle mostly on levels I and II. But level III represents such beliefs which we call modern ones and which we would like our future teachers to have. Therefore the aim of the mathematics education offered to those students is to promote construction of such beliefs.

Method and research question

The renewed mathematics unit had started in summer 1994. During following years I supervised a big number of practising lessons which were held both by students who have studied optional mathematics and by those who have done only compulsory courses. While following the lessons I got the feeling that those students who had studied more mathematics had better touch in their teaching.

This observation made me curious and I wanted to detect how the students themselves had experienced the relationship between their mathematics studies and their mathematics teaching.

Among qualitative research, persons' experiences are studied by using their own narration's (written or spoken). In my study I used semistructured interviews where the themes were 'what students had taught and how at their practising lessons', 'how they had felt with mathematics and mathematics teaching', 'what and how they would have liked to teach', 'what kind of attitudes the children had against mathematics' and 'what and how they would like to teach in future?'. Predominantly the study is a fenomenografical one.

In the beginning the question was: 'What mathematics?'. I had an idea that mathematics exists in different forms at different levels; there are lower, middle and higher versions of the same mathematical thing. For example, children can concretely play with a model of Towers of Hanoi but in the same time those towers include some high level mathematics. And I assumed that one can go somehow from top to down – that means to take topics taught at university level and find out how those topics are used at primary school.

Based on that assumption, I interviewed ten students who had done optionally 15 weeks of studies in mathematics and done their last teaching practice in autumn 1996, and asked them 'which parts of that mathematics they had learned on courses, they found to be useful when they taught at school?'.

Results

When listening through interviews, I was first disappointed because I found no such things that I had been looking for. But after analysing interviews more carefully I felt that I had found something else: First, the use of concrete and visual representations was something which students had found to be very helpful, useful and worth for using also in their own teaching. Secondly, concerning to some contents, students were more conscious of their own cognition's and also of children's cognition; the difficulties in their own learning when studying binary operations in different number systems had showed them what kind of problems children might have when they study binary operations in ten based number system.

And one thing, which every interviewed student mentioned, was the course 'empirical geometry'. They told that teaching and learning on that course departed from all other mathematics lessons they had seen before and was something which they would like to use also in their own future teaching; On that course teaching (and learning) usually began so that the teacher gave students a problem or exam to solv. Students worked in little groups and found different kind of solutions. Then every group introduced its own solutions and these different solutions were discussed together. Teacher acted like a chairman or like an expert and every solution was checked and evaluated.

So I got a feeling that it was more of a question 'how they had learnt mathematics' or 'what kind of topics about mathematics' than 'what mathematics'. And I made a conclusion that I have to change my research to be 'what kind of mathematics?'.

Discussion

The rest of the study is done by making acquaintance with earlier research made among primary school teachers and prospective primary school teachers (Ball 1991; Borko et al. 1992; Castro-Martinez and Castro-Martinez 1996;

Lindgren 1995 and 1997; Llinares and Sánchez 1996) and by trying to find out what is included in teachers mathematical know-how (Ernest 1989b, Peterson 1988, Schulman 1986, Turner-Bisset 1999).

I was satisfied when I noticed that these earlier results affirmed my results. For example Ball's study (1991) where she describes teacher students' know-how concerning place value shows that prospective teachers have difficulties to really understand what happens in simple mathematical algorithms – what is the mathematics behind mechanical calculations. The studies of Llinares and Sánchez (1996) and Castro-Martinezs (1996) for their part establish that student's knowledge of representations; concrete, visual and verbal are quite inadequate for prospective teachers.

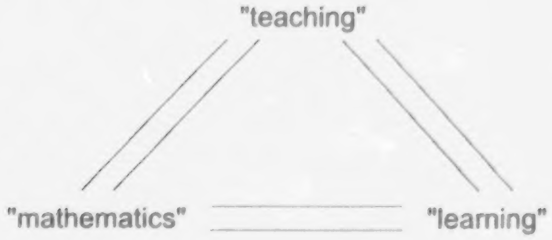
From what is said about teachers' mathematical knowledge for teaching I like to mention two things which, after my mind, tell in compact way the most essential. First is Shulmans' concept 'pedagogical content knowledge' which "includes the most regularly taught topics in one's subject area, the most useful forms of representations of those [content] ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others". It also includes an understanding of what makes the learning of specific topics easy or difficult: "the conceptions and preconceptions that students for different ages and backgrounds bring with them to learning" (Schulman 1986, 9).

Secondly Peterson (1988) sums up briefly teacher's mathematical know-how when she argues that in order to be effective, teachers of mathematics need three kinds of knowledge:

a. how pupils think in specific content areas,
b. how to facilitate growth in pupils' thinking, and
c. self-awareness of their own cognitive processes.

Peterson does not ignore the content knowledge necessary to teach, but argues convincingly that this knowledge must be regarded in relation to the three categories she has identified. Mathematics knowledge isolated from children's cognition and teachers' metacognition does not appear important to her.

Thus teachers' mathematics is mathematics which is very closely related with the teaching and learning of mathematics. Situation can be illustrated with the following simple triangle:



Implications

Since teachers' mathematics is tightly related with teaching and learning, it is reasonable to learn it in this same context. Thus the mathematics which is offered to future teachers should start from the contents taught at school, but those contents ought to be approached from the 'teacher's perspective', related to the problems of teaching and learning of those contents. This kind of mathematics is here called didactical mathematics.

The ways of teaching and learning should be the same what we expect our students to use with their pupils in future. Thus students themselves must engage in mathematical investigations. The view of teaching for understanding that begins to develop at Level 1 is transposed to a view that understanding grows out of engagement in the very processes of doing mathematics. The physical and pictorial representations are regarded as a provision both for doing tasks carefully designed by the lecturer and for generating non-standard procedures by the students. The role of the lecturer is to steer students' thinking in mathematically productive ways. The questions intend to stimulate, guide, and focus students' thinking. The students are given opportunities to express their ideas and the lecturer listens to and assesses their reasoning.

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The Metalevel of Cognition-Emotion Interaction

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The focus of this paper is in the building of theory; more specifically, the metalevel of emotion and cognition. Several authors have presented different, overlapping concepts that parallel metacognition, and extend the idea of meta-level of mental processes onto the affective domain. Based on a literature review, the metalevel is in this paper reconceptualised by dividing it into four domains. The four domains are 1) metacognition (cognitions about cognitions), 2) emotional cognition (cognitions about emotions), 3) cognitive emotions (emotions about cognitions), and 4) meta-emotions (emotions about emotions).

Introduction

The motive for this theoretical paper stems from my efforts to make sense how students' attitudes towards mathematics and their beliefs about mathematics develop (Hannula 1997, 1998a, b, 1999, submitted). Within this domain the theory is under development, and experts lack agreement on how to define the concepts on this field. For example, Furinghetti & Pehkonen (1999) did not find agreement among 18 experts on any of the nine different characterisations used for concepts on this field. Interplay of cognition and emotion seems to be significant to the development of beliefs and attitudes. This paper aims to provide a conceptual framework for the complex interaction of cognition and emotion.

Just as there is no general agreement on the terminology of attitudes and beliefs, the concepts around the psychology of emotions are also defined in the literature in a variety of ways. I prefer to use the term emotion in the sense of an emotional state. Unlike some other researchers I do not restrict the term emotion to intensive, 'hot' emotions. Following the terminology by Buck (1999), emotions in this paper refer to certain aspects of the psychological-physiological state of the person that often have low intensity and that are not always observable by self or others. Emotion and cognition are seen as highly integrated modes of information processing that regulate human behaviour.

Although mathematics sometimes is described as cold logic, emotions have an important role in learning and doing mathematics. Research on this field has been reviewed for example in McLeod (1994). This paper examines the self-referential aspects of human mind – emotions and cognitions one has about oneself – in the context of doing and learning mathematics. Schoenfeld (1985) brought the idea of metacognition to the focus of research on mathematical thinking. Later, several researchers have extended the idea of self-reference into emotional sphere. In this paper, I shall describe six of them. Unfortunately these concepts are either too broad for fine-grained analyses or too specific for analysing all self-referential aspects of human mind and their interaction. Furthermore, these concepts are partially overlapping and incompatible.

In order to reconceptualise the metalevel of human mind; a distinction will be made between emotion and cognition as two aspects of human mind. However, these borderlines do not separate one part from another. Rather, they are seen as two sides of the same coin. Using emotion and cognition as the central concepts, the self-referential aspects of human mind will be divided into four categories: cognitions about cognitions (metacognitions), cognitions about emotions (emotional cognition), emotions about cognitions (cognitive emotions), and emotions about emotions (metaemotions).

Mathematics and emotions

Research on mathematical thinking has mainly concentrated on the cognitive processes of the individual (e.g. Schoenfeld 1985; Silver 1987; Presmeg 1999). Although people during problem solving often encounter intensive emotions of frustration and/or joy, this has usually been regarded as somewhat peripheral to the actual problem solving. Although affective issues in mathematics learning have also been studied extensively the method has mostly been surveys that describe students' liking of mathematics, degree of math anxiety, self-efficacy beliefs etc. Few studies focus on the connections between affect and mathematical thinking.

Some researchers have analyzed emotional processes in problem solving. Mandler's constructivist approach (e.g. Mandler 1989) sees emotions as initiated by a gut reaction to a discrepancy of an expected schema, which is followed by cognitive analyses. According to Mandler, emotion is always expressing some aspect of value. Although Mandler's theory has been important on the field of mathematics education, it is based on an overly simplistic theory of emotions and it does not properly incorporate the influence of less intensive emotional states.

Goldin (1988, 2000) presented "affective pathways" as a framework for the dynamics of affective domain in mathematical problem solving. These pathways are established sequences of states of feelings that interact with cognition, and suggest strategies, during a problem solving process. Affects are not merely 'noise' of human behavior in problem solving, but a representational system parallel to, and crucial for, cognitive processing. This affective representational system was later divided into four facets of affective states, which interact on the individual level: emotional states, attitudes, beliefs, and values/morals/ethics (DeBellis and Goldin 1997). Other researchers (Carlson 2000; Schoenfeld 1985, 122–125) have documented emotions and managing emotional responses as characteristic for professional mathematicians' problem solving behaviour.

Previously defined self-referential concepts

The intention of this paper is to describe a conceptual framework for self-referential aspects of human mind. That includes processes that either control mental processes or code important information about (e.g. monitor) mental processes. Next we shall briefly describe six concepts that other researchers have proposed for this domain: metacognition, meta-affect, metaemotion, metamood, self-awareness, cognitive emotions, and cognitive affects.

Metacognition is an established concept in the domain of mathematical thinking, and it has been typically defined as (a) knowledge about cognitive processes, and (b) the steering of one's own cognitive processes (e.g. Simons 1994). Schoenfeld (1985, 137, 143) stated that metacognition, and control in particular, is "a major determinant of problem-solving success or failure". Few authors recognise that the usual definition includes some emotions as part of metacognition, since emotions direct attention and bias cognitive processing (summary by Williams, Watts, MacLeod & Mathews 1988, in Power & Dalgleish 1997, 73).

DeBellis and Goldin (1997) extended the idea of metacognition to the emotional sphere. They defined meta-affect as "(a) emotions about emotional states, and emotions about or within cognitive states, and (b) the monitoring and regulation of emotions". This is a broad definition, which includes several different phenomena.

Also Hooven, Gottman & Katz (1995) define their concept of meta-emotion as paralleling meta-cognition. Reflecting their research question these feelings about feelings include interpersonal aspects: "parents' awareness of specific emotions, awareness and acceptance of these emotions in their child, and their coaching of these emotions in their child." This definition partially overlaps with DeBellis' and Goldin's meta-affect. It includes the aspects of monitoring and regulation of emotions, and also emotions about emotional states. However, the idea of other persons is clearly an extension of the concept of meta-affect.

Both meta-affect and meta-emotion include the aspect of awareness of emotions. Some other researchers have used concepts that are restricted to this aspect. Metamood has been defined as awareness of one's own emotions (mentioned in Goleman 1995, reference not given) and self-awareness as an ongoing attention to one's internal states, "an interested yet unreactive witness" (Goleman 1995). The latter concept, obviously, includes also metacognitive processes.

Baron-Cohen, Spitz & Cross (1993), defined cognitive emotions as reactions in a situation produced by one's beliefs (expectations). Their study concerned surprise. Also Buck (1999) defines cognitive affects (curiosity, interest, surprise, boredom, burnout) to involve expectancies of reward or punishment.

Theoretical background

This article makes a distinction emotion and cognition as two aspects of human mind. These two have some phenomenological differences that make this split reasonable. Cognition is neurone-based information processing, whereas emotions include also other kinds of physiological reactions. However, this splitting is only an analytical tool, and the interaction between cognition and emotion is so intense, that neither of the two can be fully understood in separation from the other.

The present view of cognition shall be outlined relatively briefly. Emotions need significantly more elaboration for two reasons. Firstly, there is less agreement on this field over the theories of emotion or even the definitions for emotion. Therefore the present standpoint can not be explained with such brevity, as is the case with cognition. Secondly, the emotions are more central to the present paper.

The present approach to cognition has its background in the cognitive science. Cognition is seen in a connectionist perspective, emerging from neural activity in the brain. It consists of several parallel subsystems, and most of its processes are unconscious. Its three basic processes are pattern recognition, categorisation, and association. Yet, it can learn complicated rules, procedures, and concepts. Lakoff and Núñes (2000) present some ideas as to how this is possible.

Although researchers have not agreed upon what they mean with emotions, there is agreement on certain aspects. First and foremost emotions are seen in connection to personal goals: they code information about progress towards goals and possible blockades as well as suggest strategies for overcoming obstacles. Emotions are also seen to involve a physiological reaction, as a distinction from non-emotional cognition. Thirdly, emotions are also seen to be functional, i.e. they have an important role in human coping and adaptation. (E.g. Buck 1999; Lazarus 1991; Mandler 1989; Power & Dalgleish 1997)

The two main issues that researchers have not agreed upon are the borderline between emotion and cognition, and the number of different emotions. The present approach adopts the controversial standpoint by Buck (1999), that emotions are always present in human existence. However, only when the intensity of emotions is high enough, they may become observable by self and others. According to Buck emotions have three mutually independent readouts: adaptive-homeostatic arousal responses (e.g. releasing adrenaline in blood), expressive displays (e.g. smiling), and subjective experience (e.g. feeling excited). Emotions that are primarily self-regulative do not always have an expressive display and if the function of the emotion is to regulate cognition, adaptive-homeostatic arousal might also be weak. Regarding the number of emotions, this paper adopts the basic approach that there are only a few basic emotions (happiness, sadness, fear, anger, disgust, and surprise), and the more complex emotions are based on these (Buck 1999; Power & Dalgleish 1997). For example, jealousy could be a mixed emotion of fear, anger, and disgust towards someone, who one perceives to threaten a valued relationship.

Metamind

The metalevel of the mind consists of emotions and cognitions that are about our own mind. This metalevel of mental processes is the focus of this paper. Our conscious awareness and control of our thoughts and feelings are part of this metalevel. However, there are also unconscious cognitive states that monitor and regulate mental processes. The approach chosen here for the 'aboutness' of emotions is through goals. While emotions are always related to goals, we can distinguish mental goals: desired cognitive-emotional states. When we approach these goals or we perceive obstacles related to these goals, our emotional state changes. These emotions are about our mind and thus part of the metalevel.

Because emotions and cognitions are by their nature quite different, it is useful to distinguish them both as meta-level processes in mind, and as objects of those processes. Thus the metalevel of mind will be divided into four aspects:

1) metacognition (cognitions about cognitions),
2) emotional cognition (cognitions about emotions),
3) cognitive emotions (emotions about cognitions), and
4) meta-emotions (emotions about emotions).

Within each of these, we can separate the aspects of monitoring and control.

Let us take a hypothetical student to illustrate these different aspects. A student, who is blocked during a task because he does not remember a necessary formula, would experience frustration (cognitive emotion). This frustration could make him anxious (meta-emotion). He might be aware of these emotions and he could even consciously try to calm down (emotional cognition). Then he could decide to use a different strategy (metacognitive regulation), and proceed with the task. I will now elaborate on each of these aspects, and relate them to previously defined concepts.

1) The present paper slightly sharpens the definition of metacognition by restricting steering to cognitive steering. This definition does not include direction of attention and bias of cognitive processing that is caused by emotions.

2) Emotional cognition is the sister of metacognition. It includes the subjective knowledge of one's own emotional state and emotional processes. However, the subjective *experience* of an emotion fits more appropriately to the domain of emotions, although it, too, is a kind of cognition (Buck 1999). Of the previously defined concepts, metamood is the awareness-aspect of emotional cognition. On the other hand, self-awareness extends the monitoring aspect to include the monitoring of cognitions as well. Furthermore, meta-affect and meta-emotion include emotional cognition as part of the broader concept.

Students are aware about the different emotions they have in different situations and they even know of their typical emotional reactions in mathematics class. The subjective knowledge of one's own emotions is the basis for emotional expectations in different situations, and thereby it directs the approaches one has towards mathematical situations. For example, some cases of mathematics anxiety presumably are based on the expectancy to experience an unpleasant emotion. Likewise, positive expectations are the basis for some students' motivation to engage in mathematical situations. Elsewhere (Hannula 1998, 2000, submitted), I described a case study of one student, whose attitude towards mathematics changed dramatically. The reasons she gave for her more positive attitude were that mathematics is 'more fun', because she had 'been understanding more'.

Emotional cognition also includes the direct cognitive control of emotions, typically inhibition. For example, I had observed one anxious having problems at one point during the lesson. When we discussed that situation after the lesson, almost an hour later, she started to cry because she felt unable to understand anything during the lessons. Yet, she had kept herself calm in front of her classmates. (Hannula 2000)

Emotional cognition and metacognition are the two kinds of cognitions one can have about one's own mind. These are the aspects of mind that subjects in interviews can tell about themselves. Therefore, these concepts are of interest with respect to methodology of mathematics education.

3) Emotions exist in relationship with goals. Sometimes goals may be cognitive, as was the case in Schoenfeld (1985), in which the professional mathematician wanted to remember one geometrical construction. On his not-so-straightforward way towards this goal, he experienced surprise, chagrin, and pleasure (p. 122–125). In this paper, emotions that are related to cognitive goals are called cognitive emotions. Cognitive goals may be explicit, like when one wants to remember a fact or a procedure, or when one tries to solve a mathematical problem. Sometimes the goal may be vague, like 'to understand' a topic.

Cognitive emotions seems to be the focal point in Goldin (1988, 2000), where he describes affective pathways as controlling cognitive processes during problem solving behaviour. Later, this idea is extended to a broader concept of meta-affect. The concepts of cognitive emotions (as used by Baron-Cohen et al 1993) and cognitive affects (Buck 1999) seem, at first; different from the one used here. However, they are related. Surprise (as a cognitive emotion) is the emotional reaction when the cognitive goal of creating apt expectations is not met. Similarly, as Buck points out, all cognitive affects in his list are related to learning goals.

4) Meta-emotions are emotional reactions to one's own emotions. These meta-emotions code important information about the appropriateness of the emotion in question and they also control that emotion. On an interpersonal level, this aspect is the centre of the concept of Hooven et al.'s (1995) meta-emotion. In this paper the concept is restricted to individual level. Also meta-affect as a broad concept includes this aspect (DeBellis & Goldin 1997). Presumably all humans share the goal to experience pleasure and avoid unpleasant emotions. Humans have also the capacity to tolerate unpleasant emotions if reward of pleasure is to be expected later. For example, successful problem solvers are prepared to tolerate frustration on their way towards solution. There are, however, different norms and individual coping strategies concerning emotions. Therefore, the same emotion may be more stressful for one individual than the other.

Within a context of mathematical problem solving, students may use emotions to code information, and these emotions are accepted more easily than 'real' emotions. For example, DeBellis & Goldin (1997) reported how a student expressed dislike, while she described the difficulty to divide a round cake in three equal pieces. This dislike, however, did not cause any discomfort for the student. The meta-emotional reaction to this emotion in this context was neutralising.

Concluding remarks

Metacognition has been accepted as an important aspect of mathematical thinking. Recent research indicates also the important role of emotions in mathematical thinking. To combine these two approaches and focus on the metalevel of emotion-cognition interaction was a logical next step. In this paper this domain of research was divided into four categories that all are phenomenologically different. This approach provides a new way to look at the emotional-cognitive processes during mathematical thinking and learning, and it has been useful in understanding of the development of attitudes and beliefs (Hannula, submitted).

The distinctions made here are important also methodologically. In interviews students can tell only about the aspects of their mind they are aware of. Therefore self-report based research can reach only metacognition and emotional cognition. However, emotions can be reached directly through measuring adaptive-homeostatic response (e.g. Isoda & Nakgoshi 2000) or observing facial expressions (e.g. DeBellis & Goldin 1997).

There are also some implications for teaching practice. If we accept that cognitive emotions (e.g. surprise, interest, and frustration) have an important role in the control of cognitive processes, then educating about these emotions is important. Relevant emotional education would include the awareness of these emotions and knowledge of their importance to thinking skills (emotional cognition), acceptance of emotions as part of cognitive processes (meta-emotion), and finally, increased control over them.

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Problem Solving in Chemistry and Science Education

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This paper reviews some important contributions to problem solving in chemistry and science education. The subject of stoichiometry calculations is separately treated. In the past, attention was centred on the methods and procedures used by both students and experts when solving problems. Subsequently, emphasis has been shifted to the role of psychometric factors. Working-memory capacity is crucial in the Johnstone-El Banna predictive model; while disembodying ability, "chunking" of the problem into familiar parts, as well as the so-called "noise" which may be present in the problem have an effect too. We have stated a number of necessary conditions for the successful application of the Johnstone-El Banna model, and have tested the model in the case of organic-synthesis problems. In complicated problems, such as molecular-equilibrium ones, important becomes the logical structure of the problem. Extensive practice can turn the problems into algorithmic exercises; developmental level plays an important part then. Finally, mention is made of a recent innovation of applying methods of complexity theory to problem-solving data.

Introduction

Problem types

There is a vast variety of problems: closed problems, with one answer; open problems, which can have more than one answer and for which data may not be supplied; problems that can be solved by pencil-and-paper or by the computer; problems that need experiment in order to be solved; and real-life problems versus scientific problems or even thought problems.

At the outset, a distinction must be made between problems and exercises, with the latter requiring for their solution only the application of well-known and practised procedures (*algorithms*). The skills that are necessary for the solution of exercises are as a rule *lower-order cognitive skills* (LOCS). On the other hand, a real/novel problem requires that the solver must be able to use what has been termed as *higher-order cognitive skills* (HOCS) (Zoller 1993; Zoller & Tsaparlis 1997). However, the degree to which a problem is a novel problem or an exercise depends on the student background and the teaching (Niaz 1995a); thus, a problem that requires HOCS for some students may require LOCS for others in a different context.

A more thorough classification of problem types has been made by Johnstone (1993) and is reproduced here in Table 1.

TABLE 1. Classification of problems (Johnstone 1993).

Type	Data	Methods	Outcomes/Goals	Skills bonus
1	Given	Familiar	Given	Recall of algorithms
2	Given	Unfamiliar	Given	Looking for parallels to known methods
3	Incomplete	Familiar	Given	Analyses of problem to decide what further data are required.
4	Incomplete	Unfamiliar	Given	Weighing up possible methods and then deciding on data required.
5	Given	Familiar	Open	Decision making about appropriate goals. Exploration of knowledge networks.
6	Given	Unfamiliar	Open	Decisions about goals and choices of appropriate methods. Exploration of knowledge and technique networks.
7	Incomplete	Familiar	Open	Once goals have been specified by the student these data are seen to be incomplete.
8	Incomplete	Unfamiliar	Open	Suggestion of goals and methods to get there; consequent need for additional data. All of the above skills.

Types 1 and 2 are the "normal" problems usually encountered in academic situations. Only type 1 is of the algorithmic nature (exercise). Type 2 can become algorithmic with experience or teaching. Types 3 and 4 are more complex, with type 4 requiring very different reasoning from that used in types 1 and 2. Types 5 to 8 have open outcomes and/or goals, and are very demanding. Type 8 is the nearest to real everyday problems.

Overview of problem-solving research

In the earlier studies of problem solving in the physical sciences, attention was centred on the methods and procedures used by both students and experts when solving problems. Subsequently, emphasis has been shifted to the role of psychometric factors. Working-memory capacity is crucial in the Johnstone-El Banna predictive model, while dis-embedding ability, "chunking" of the problem into familiar parts, as well as the so-called "noise" which may be present in the problem have an effect too. In complicated problems, important becomes the logical structure of the problem, while developmental level plays an important part in algorithmic quantitative problems. We have stated a number of necessary conditions for the successful application of the Johnstone-El Banna model, and have tested the model in the case of organic-synthesis problems. Our attention is then focused on molecular-equilibrium problems and their logical structure. Finally, mention will be made of a recent innovation with the application of complexity theory to our data. An earlier review has been presented at the 4th *ECRICE* (Tsaparlis 1993 and 1997).

Review of some earlier work

Methods and procedures of problem solving

A number of researchers (Simon & Simon 1978; Larkin & Reif 1979; Larkin 1980; Reif 1981) have studied the differences between expert and novice problem solvers. The basic differences were:

(a) the comprehensive and complete scheme of the experts, in contrast to the sketchy one of the novices

(b) the extra step of the qualitative analysis taken by the experts, before they move into detailed and quantitative means of solution.

Various suggestions have been made for the improvement of the problem-solving capabilities of students. Mettes and co-workers (Mettes et al. 1980; Kramers-Pals & Pilot 1988) have proposed useful algorithmic methods, in which they include the steps that characterise expert solvers, viz. the qualitative estimation (prediction) of the result, the check of the validity of the equations and relations, as well as the evaluation of the result. Genya (1983) has proposed the so-called sequences of problems of gradually increasing complexity, in which qualitative problems appear at the beginning.

To become good solvers, students must be given ample practice in solving problems. This will not give them just practice, it will develop confidence. The following should be constantly in the mind of every teacher (Frazer 1985)

> "Students need to be confronted with problems carefully selected to provide tasks, which are not beyond their level of knowledge and skills. Our intention must be that students should succeed and not to fail."

Further, students who have obtained good answers should not be ignored by the teacher; they also need guidance and help. The problems must be challenging, real problems, selected from the chemical literature, the solution of which will not be obvious to the student, and yet the necessary information and reasoning is likely to be within students grasp. Frazer has tested with both university and high-school students a method of problem solving in tutorials, in which emphasis is given to peer group teaching (Frazer 1982). One problem is given at a time. Students work first on their own, then in-groups of four.

Stoichiometry calculations

Stoichiometry calculations are central and unique to chemistry. They are difficult for many students, not so much because of their mathematical complexity, but mainly because of the abstract concepts involved. A number of methods for tackling stoichiometry calculations have been suggested. They can be distinguished grossly into logical and rote methods. As logical methods we assume the proportion method, the common sense method (Navidi & Baker 1984), the unit-base method (Beichl 1986), and the step-by-step method on a "road map" (Fast 1985). As rote methods we assume the rule-of-three method, the factor-label method, and the conversion matrix method (Berger 1985).

Schmidt (1997) has reviewed both the methods and the research on stoichiometry calculations.

The proportion method is considered difficult for students because it involves proportional reasoning, formal-operations ability. The factor-label method is based on dimensional analysis. According to Navidi and Baker (1984), the common sense method should be used first, while the factor-label method should be postponed. On the other hand, Gabel and Sherwood (1983) compared the effectiveness of four methods for teaching stoichiometry problems and concluded that the factor-label method was the most effective, while a method based on proportions was the least effective.

The rule-of-three method is used, for instance, in Germany (Schmidt 1997) and Greece. To demonstrate its use in Greece as a rote method, consider the following problem:

> *How many grams of NO will be produced when 20.0 g of* O_2 *has reacted according to the equation* $4\ NH_3 + 5\ O_2 \rightarrow 4\ NO + 6\ H_2O$?
>
> *Solution:*
>
> From 5 mol O_2 that is from 160.0 g O_2, 4 mol NO, that is 120.0 g NO are produced
> From 20.0 g $O_2 \rightarrow$ x g NO are produced
>
> $$x = (120.0) * (20.0)/160.0 = 15.0$$

We are of the opinion that the use of rote methods in the teaching of stoichiometry problems should be avoided in the first encounter of students with chemistry (Tsaparlis 1988). Instead, a logical method should be preferred and in particular one that makes the calculations easy to understand. Proportions should be avoided in the first place. Later, when students are more comfortable with chemistry, one can switch to the logic of proportions, and even to a rote method. This opinion is in accord with the findings of research (Schmidt 1990 and 1994; Zarotiadou, Georgiadou & Tsaparlis 1995). Schmidt (1997) recommends that initially problems should be used that can be solved easily mathematically as well as by "pure logic".

The Johnstone-El Banna predictive model and related neo-piagetian studies

Basic concepts

Problem-solving research has entered a new phase with the consideration of the importance of information processing. Each subject has a certain working memory capacity, X, while each problem has a Z-demand. One definition of demand is "*the maximum number of thought steps and processes which have to be activated by the least able, but ultimately successful solver, in the light of what he has been taught*" (Johnstone & El-Banna 1989). The Johnstone-El Banna model (Johnstone & Kellet 1980; Johnstone 1984; Johnstone & El-Banna 1986 and 1989) is a predictive model which states that a subject will not be successful in solving a problem, unless the problem has a Z-demand which is less or equal to the subject's X-capacity.

Related to the working memory capacity X is the mental capacity or M-capacity, which derives from Pascual-Leone's neo-piagetian theory. A mental power, or M-space or M-capacity, is attributed to each subject, which is further distinguished into the maximum available M-capacity or structural M-capacity (M_s), and the actually mobilised M-capacity or functional M-capacity (M_f). Niaz has reported significant correlations between M-capacity and M-demand (which is synonymous to Z-demand) in numerous studies (e.g. Niaz & Lawson 1985; Niaz 1987, 1988a, 1988b, 1989a, 1989b, 1989c, 1991, 1994a and 1996). Even small changes in M-demand can lead to working-memory overload; further, the manipulation of the logical structure (that is the degree to which the problem requires formal operational reasoning) could also lead to significant changes in the student performance (Niaz & Robinson 1992). Niaz (1994b) has also studied the mobility-fixity dimension.

Both working memory and M-capacity refer to some kind of hypothesised limited capacity mental resource which can be applied to learning and problem solving, and the two steps appear to be used interchangeably (Niaz & Logie 1993; Vaquero, Rojas, & Niaz 1996).

Blocking mechanisms in problem solving and necessary conditions for the Johnstone-El Banna model

In a number of cases, the Johnstone-El Banna model may be violated. Using non-numerical organic chemical-synthesis problems of Z-demand of two, we have examined mechanisms that block the solution (Tsaparlis 1994, 1995, and 1996). Table 2 has examples of such problems.

TABLE 2. Some organic synthesis problems with Z-demand of two.

Suggest routes for the synthesis of the following organic compounds, using as reactants the organic compound given in each case plus any inorganic compounds (e.g. water, acids, bases, salts, metals) and any required conditions (e.g. heating, light, catalysts).

- Isopropyl bromide, $CH_3CHBrCH_3$, from isopropyl chloride, $CH_3CHClCH_3$.
  [Answer:
  $CH_3CHClCH_3 + NaOH \rightarrow CH_3CH{=}CH_2 + NaCl + H_2O$
  $CH_3CH{=}CH_2 + HBr \rightarrow CH_3CHBrCH_3$]
- Racemic mixture of the D and L optical isomers of galactic acid, $CH_3{}^*CH(OH)COOH$, from propionic acid, CH_3CH_2COOH
  [Answer:
  $CH_3CH_2COOH + Cl_2$ (in the presence of red P or light) $\rightarrow CH_3{}^*CHClCOOH + HCl$
  $CH_3{}^*CHClCOOH + OH^- \rightarrow CH_3{}^*CH(OH)COOH + Cl^-$]

These problems not only exclude numerical or algebraic calculations, but also have a unique chemical logical structure. The number of logical schemata involved in a problem may be the main factor in determining the difficulty of the problem, overriding its Z-demand (Niaz & Robinson 1992). The subjects were first-year chemistry students. The following blocking mechanisms have been verified:

(1) The lack in the subject's repertoire of even a single step in the solution.
(2) The non-equivalence of the partial steps of the solution.

(3) The blocking that occurs when subjects fail to solve a problem, even if they have the partial steps available.

(4) The presence of "noise" in the problem (Johnstone & Wham 1982). By "noise" one means information that is in the problem, but has no actual involvement in the solution process. It was found that predominantly students with low working memory capacity are affected by the "noise" (see also Johnstone & Al-Naeme 1991).

Our findings, together with other bibliographical evidence have led us to state the following necessary conditions that must be observed for the successful application of the Johnstone-El Banna model to problem solving in science education. (Tsaparlis 1994, 1995 and 1998).

1) The partial steps must be *available* in long-term memory, and *accessible* from it.
2) The partial steps must be easily accessible. The organisation and the connections in long-term memory are crucial for the accessibility of knowledge (Johnstone 1991).
3) The logical structure of the problem must be simple; otherwise, developmental level may play a more important role than working-memory capacity (Niaz & Robinson 1992). The best situation is when only one operative schema exists in the problem. The organic-synthesis problems, which exclude numerical or algebraic calculations, are one example of such simple-structure problems.
4) No "noise" should be present in the problem statement. The presence of "noise" may lead to working-memory overload.
5) The degree of field independence (disembodying ability) may have an effect. Low and, to a lesser extent, intermediate working-capacity subjects who are field dependent may experience a working-memory overload, caused by "noise" (Johnstone & Al-Naeme 1991).
6) Familiarity with the problem, as well as chunking of the problem into familiar chunks, result in a reduction of the Z-demand of the problem. Therefore, the model must be valid in the case of actual problems and not of exercises.

Failure to observe these conditions may lead to data that will not follow the model, or it will not follow it closely. In our opinion it is such failure which has led to the invalidation of the model in the work of Opdenacker et al. (1990), where highly competent medical students who were likely to be field independent and who had many learning strategies were involved. These students never overloaded, and this explains the lack of fit of their performance with the model.

Operation, validation and usefulness of the model: The case of organic-synthesis problems

Predictive-explanatory models are very useful for understanding the factors that influence the ability of students to solve problems. They place on a rigorous and quantitative basis both the factors that affect the general problem-solving ability of students, and the structure of the problems themselves. We have stated above a number of necessary conditions for the application of the Johnstone-El Banna model. In a recent publication (Tsaparlis & Angelopoulos 2000) we used again organic-synthesis problems to test the validity of the model. Problems with varying Z-demand of 2, 3, 4, 5, 6, 7 and 8 were used. Table 3 has some examples. For the degree of realisation of the above stated necessary conditions see the original paper (Tsaparlis & Angelopoulos 2000).

TABLE 3. Examples of organic-synthesis problems with varying Z-demand.

- Acetaldehyde, CH_3CHO, from ethene, $CH_2=CH_2$.
- 1,2,3-trichlorobutane, $CH_3CHClCHClCH_2Cl$ from ethyne, $CH\equiv CH$.
- 2-cetopropionic acid, $CH_3COCOOH$, from acetone, CH_3COCH_3.
- 2-methylbutanoic acid, $CH_3CH(CH_3)COOH$, from acetaldehyde, CH_3CHO.
- Formaldehyde, $HCHO$, from sodium acetate, CH_3COONa.
- Propyne, $CH_3C\equiv CH$, from 2-propanol, $CH_3CH(OH)CH_3$.
- Pentane, $CH_3CH_2CH_2CH_2CH_3$, from 1-propanol, $CH_3CH_2CH_2OH$.
- 3-hydroxypropionic acid, $HOCH_2CH_2COOH$, from formaldehyde, $HCHO$.

Our subjects were upper-secondary students in their final school year, preparing to take entrance examinations for tertiary education (age 17–18) who attended three public urban schools (N = 191) in the greater Athens region. In addition, there was another sample of students (N = 128) from two urban special private schools ("*frontisteria*"). These students have not received previous instruction (practice) to this type of problems. On the other hand, because most public-school students attend at the same time *frontisteria*, a considerable number of them had previous experience with the problems.

Figure 1 shows the performance in terms of completely right or wrong, that is the facility value (FV) of public school students (N = 191) in problems of varying Z-demand, according to the measured working memory capacity of the students. Figure 2 shows the corresponding information for *frontisteria* students (N = 128). The operation of the Johnstone-El Banna model is evident in both cases, but it is more pronounced in the case of *frontisteria* students. It is clear that students with a measured working memory capacity X = 4 do well on problems up to a Z-demand of 4, but decline rapidly for questions with a Z value equal to or greater than 5. Similarly, those with X = 5 do badly for questions with Z > 6. A similar pattern appears for those with X = 6 and X = 7. In the case of public schools (Figure 1), there is a less dramatic fall for the X = 7 group, showing that their previous experience and their chunking skills may be operating. On the other hand, *frontisteria* students with X = 7 approached the ideal prediction of 100% success up to Z < X, followed by a large drop when Z > X.

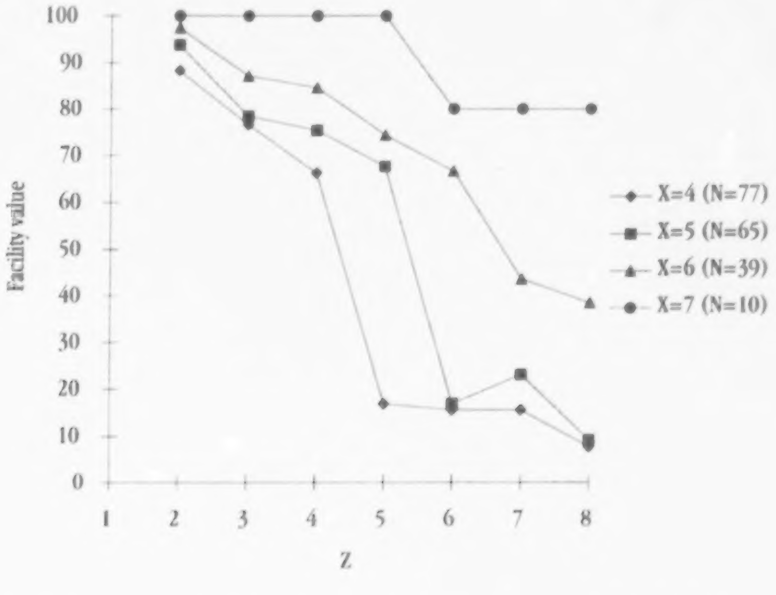

FIGURE 1. Public schools (N = 191). Marking: Successful or failed.

Summarising, we find that as Z increases, there is a general *slow* fall in performance, but no difference is significant until Z *just* exceeds X. The fall is significant at the "break" point, but, after that, a gentle slow fall occurs again, in which none of the differences is significant. In each case, the significant fall is when Z = X + 1. Finally, a number of students, notably higher for higher information-processing capacity, can use chunking devices, and this keeps performance from eventually falling to zero.

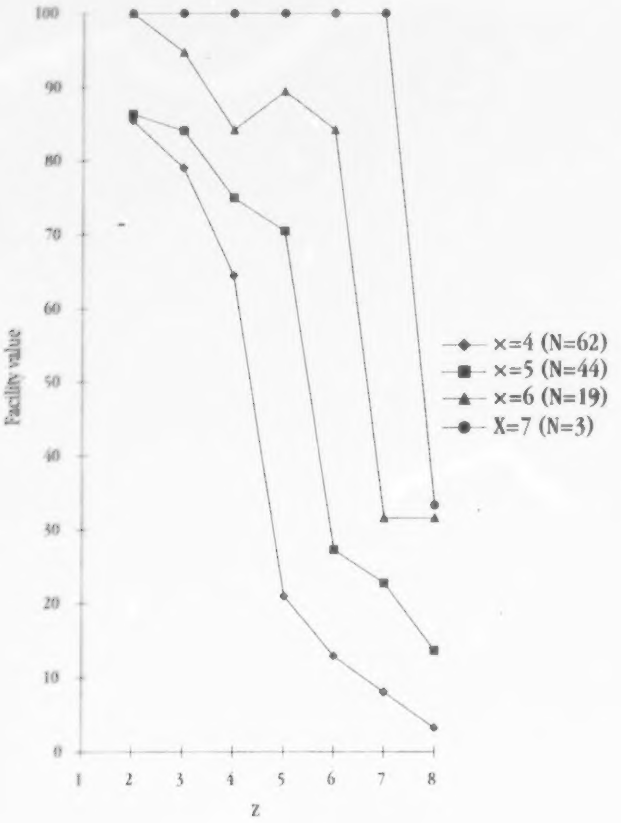

FIGURE 2. Frontisteria (N = 128). Marking: Successful or failed.

The main conclusion is that it can be predicted that students who have a high working memory capacity (at least of six) [and at the same time are not field-dependent (see Tsaparlis & Angelopoulos 2000)] are capable of high information processing, having an advantage in problem solving. Results obtained in this study, together with those from other studies (Niaz 1995a and 1995b; Tsaparlis, Kousathana & Niaz 1998), suggest that problem solving is a very complicated process, involving more than one cognitive variable, as well as affective ones, and have important implications for instruction in problem solving.

The role of the logical structure: The case of chemical equilibrium problems

Review of research on chemical equilibrium

Chemical-equilibrium problems are among the most important, and at the same time most complex and difficult general chemistry problems. Camacho and Good (1989) studied the problem-solving behaviours of experts and novices engaged in solving chemical-equilibrium problems, and reported that unsuccessful subjects had many knowledge gaps and misconceptions about chemical equilibrium. Wilson (1994) examined the network representation of knowledge about chemical equilibrium, and found that the degree of hierarchical organisation varied, and that the differences reflect achievement and relative experience in chemical equilibrium. Gussarsky and Gorodetsky (1988) have previously reported similar findings. It is noteworthy that students use algorithmic methods without understanding the relevant concepts (Gabel, Sherwood & Enochs 1984). Niaz (1995a) has reported that students who perform better on chemical-equilibrium problems requiring conceptual understanding also perform significantly better on computational problems; he further suggested that solving computational problems before conceptual problems would be more conducive to learning. Tsaparlis and Kousathana (1995) have reported student's common misconceptions and errors in solving molecular equilibrium problems. Finally, Pedrosa and Dias (2000) have compared student alternative conceptions about chemical equilibrium with chemistry textbook approaches to that topic.

The logical structure of the problem

Niaz and Robinson (1992) have examined the effect on student performance of the manipulation of the *logical structure* of stoichiometry problems. They reported that the developmental level of students is the most consistent predictor of success when dealing with significant changes in the logical complexity of chemistry problems, so that the number of *operative schemata* involved in a problem may be the main factor in determining the difficulty of the problem, overriding its M-demand. According to the above authors, "*the logical structure of a problem represents the degree to which it requires formal operational reasoning, that is, use of proportions, making quantitative correlations, generation of all possible combinations, controlling variables, and hypothetico-deductive reasoning*" (Niaz & Robinson 1992, p. 212).

The logical structure of a problem is specified by the number of *operative schemata* entering the problem. According to Piaget, a *schema* is an internal structure or representation, while the ways we manipulate schemata are called *operations*. In Piaget's theory, schemata are continually growing and developing rather than remaining fixed. Describing thinking at various stages thus becomes a problem of trying to define the schema (or mental structure) and the operations (or internal actions) that a problem solver is using (Niaz & Robinson 1992).

Manipulation of logical structure and of Z-demand in the case of molecular-equilibrium problems

In our work (Tsaparlis, Kousathana & Niaz 1998), we examined the effect on student performance of the manipulation of the logical structure as well as of the Z-demand of chemical (molecular) equilibrium problems. Note that we dealt only with *chemical* schemata that enter molecular-equilibrium problems, but not with *mathematical schemata* (algebraic or calculational). We have identified the following schemata of molecular equilibrium:

Schema 1. The process of establishment of the chemical equilibrium.
Schema 2. The condition of chemical equilibrium.

Schema 3. The case of gaseous systems, with use of partial and total pressures as well as of K_p.

Schema 4. The disturbance of the equilibrium and the establishment of a new equilibrium.

TABLE 4. A chemical-equilibrium problem with analysis of schemata and Z-demand.

In a vessel of fixed volume $V = 4.5$ dm^3, 198 g $COCl_2$ plus 44.8 dm^3 CO (in STP) are introduced. The mixture is heated to 100°C and let to reach the equilibrium $COCl_2(g)$ <=> $CO(g) + Cl_2(g)$. You have to calculate the equilibrium constant K_c, taking into account that at equilibrium the total pressure of the gas mixture is 82 atm, at 1000°C.

Three schemata enter here:

(1) The process of establishment of the chemical equilibrium

(2) the ideal-gas equation

(3) the condition of chemical equilibrium *(K_c)*

(1) Establishment of equilibrium

Calculation of moles of $COCl_2$: *a* mol (Step 1)

Calculation of moles of CO: *b* mol (Step 2)

	$COCl_2(g)$ =	$CO(g)$ +	$Cl_2(g)$	
initially:	a mol	b mol		(Step 3)
react (-) or produced (+):	-x mol	+x mol	+x mol	(Step 4)
at equilibrium (moles):	(a-x) mol	(b+x) mol	x mol	
at equilibrium (concentrations):	(a-x)/V mol/L	(b+x)/V mol/L	x/V mol/L	(Step 5)

(2) Ideal-gas equation

$p_{total} V = n_{total} R T$ (Step 1)

$n_{total} = p_{total} V/(R T)$ (Step 2)

Calculation of n_{total} (Step 3)

Calculation of x (Step 4)

(3) Condition of chemical equilibrium (K_c)

$K_c = (CO)(Cl_2)/(COCl_2)$ (Step 1)

$K_c = [(b+x)/V](x/V)/[(a-x)/V]$ (Step 2)

Calculation of K_c (Step 3)

Estimation of M-demand. Schema (1) of the establishment of equilibrium involves the largest number of steps (five), hence we postulate an M-demand of 5 for this problem.

Nine problems of varying number of operative schemata as well as of varying number of Z-demand were used. The number of schemata varied from a minimum of two, to a maximum of four, while within each logical schema we had a specific Z-demand, varying from 4 to 6. The structure of the nine problems was as follows (the first number stands for the number of schemata, the second number stands for the Z-demand): (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6). Table 4 has an example of a problem, with an analysis of schemata and Z-demand.

Table 5 gives the percentage mean performance in the sets of problems with the same number of schemata, as well as in the sets of problems with the same Z-demand. We note that the overall performance was relatively high; this is due to the fact that our subjects were highly motivated for this course, because it was very crucial for their university entrance examinations. We note also that there was a larger drop in performance when one increased the logical structure by adding schemata than when one increased the Z-demand. This finding is accordance with that of Niaz and Robinson (1992).

TABLE 5. Percentage mean performance of all students ($N = 154$) in the sets of problems with (a) the same number of schemata [e.g. problems (3,4), (3,5) and (3,6)], and (b) the same Z-demand [e.g. problems (2, 5), (3, 5) and (4, 5)]. Standard deviations are given in parentheses.

Problems with 2 schemata	**Problems with 3 schemata**	**Problems with 4 schemata**
92.0 (10.1)	83.4 (16.0)	71.0 (26.0)
Problems with Z-demand = 4	Problems with Z -demand = 5	Problems with Z -demand = 6
88.7 (13.2)	84.3 (18.8)	73.4 (20.0)

In addition, the correlations between four cognitive variables (developmental level, working memory, functional M-capacity and disembodying ability) and the student achievement were reported (Tsaparlis, Kousathana & Niaz 1998). It was found that all cognitive variables are quite consistent in correlating with achievement only when the logical structure is fairly complex and even when the Z-demand is relatively low. Developmental level plays the dominant part which can be accounted for by taking into account that these problems were of an algorithmic nature for the subjects of this study, because of their extended practice (the "training effect"), in accordance with the research finding of Niaz and Robinson (1992). On the other hand, working memory maintains some importance too, because there is always a need for information processing in solving the problems; however, training on task could lead to "chunking", which reduces the Z-demand of a task and thus improves student performance (Pascual-Leone et al. 1978). Finally, working memory and functional M-capacity, the two variables representing information processing, correlate significantly with all problems (except problem 2,4) at about the same level. However, it may be that the latter has a smaller power to explain variance.

Application of complexity theory to problem-solving

A novel attempt to test the Johnstone-El Banna model has been made recently by introducing non-linear methods in the treatment of problem-solving quantitative data. The work correlates the rank order of the subject achievement scores in problem solving with the working-memory capacity and shows how the effect of working memory on problem solving can be observed with means and tools of complexity theory.

A problem-solving data set consists of achievement scores. In order to apply complexity theory to this data set, *dynamics* need to be introduced. This is achieved by using the rank-order sequences of achievement, which are treated as dynamical flows. Rank-order sequences of achievements of the subjects, according to their scores, are generated, and in the place of each subject, his/her score is then replaced by the value of his/her working-memory capacity. These *dynamic* sequences when treated with tools of complexity theory may appear to posses order or disorder that characterise its fractal geometry. The characterisation of that geometry can be made by using indexes of complexity theory such as the Hurst exponent, fractal dimensions, or entropy.

The working hypothesis, which is behind the non-linear treatment, is whether there exist scale-invariant, long-range correlations in the created sequence of working memories. The study of these correlations was made by means of the *working-memory random walk* method. This method has previously been

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reported (Stamovlasis & Tsaparlis 1999; Tsaparlis & Stamovlasis 1999). Data have been taken from the achievement scores in simple organic-synthesis problems (Tsaparlis & Angelopoulos 2000).

In the hypothetical one-dimensional space of the rank order achievement scores, we take a random walk among the subjects with working memory capacity 4 and 5. This walk is named random walk 4/5. It is observed that the Hurst exponent for low values of Z-demand, 2,3, and 4 has a low value close to the surrogate exponent, which corresponds to theoretical randomness: $H=0.5$, a normal random walk. The results for the Hurst exponent are shown in Figure 3. At Z-demand 5, the value of Hurst exponent increases, showing long-range correlation, because subjects with working memory capacity 5 outscore subjects with working memory capacity 4. When the Z-demand of the problem becomes 6, the problem becomes difficult for everybody, and the sequence becomes disordered again. A similar pattern is observed for random walk 5/6.

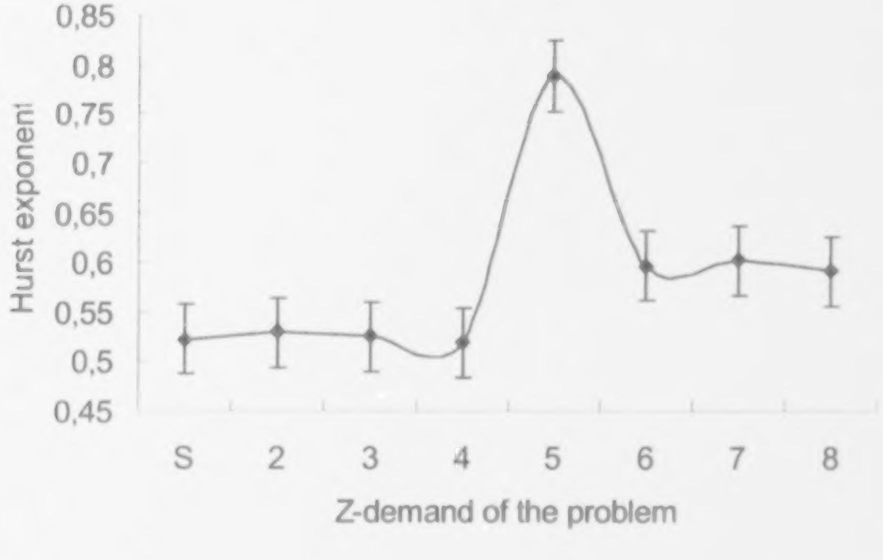

FIGURE 3. The Hurst exponent versus Z-demand of the problem for the random walk 4/5. ($\Psi(i) = -1$, for $X = 4$; $\Psi(i) = +1$, for $X = 5$). The sudden increase appears from Z-demand 4 to Z-demand 5.

A recent publication (Stamovlasis & Tsaparlis 2000a) provides further related information, while an extended version of this work will appear in the journal *Non-linear Dynamics in Psychology and Life Sciences* early in 2001 (Stamovlasis & Tsaparlis 2000b).

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Physics Education Research: Inseparable Contents and Methods – The Part Played by Critical Details

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This address centres on the interrelation that exists – de facto – between conceptual teaching goals and what are often called "methods". Three examples are taken, two from mechanics and one on colour, to illustrate this idea. In each case, the implications of research based teaching strategies in terms of conceptual content, and vice versa, are discussed. In line with this idea, the part of some aspects of practice that may seem of minor importance, but may prove critical in terms of learning effects is also analysed. A brief discussion of the implications of these views, particularly in terms of teacher training, ends the paper.

Introduction

It is often said that physics – or chemistry or math – education research is devoted to methods, the content to be taught being independently decided. A common idea among teachers at various levels is that didactics is concerned with how to teach, whereas what to teach is only a matter of selection of topics. The content matter would be unquestionable, the only decision being about what slice to select in this content, taking into account "the level of the students".

In order to support the opposite claim, I will show how an investigation concerning student's difficulties and a discussion about "what-and-how" to teach in this domain are overlapping concerns, which lead one to examine classically taught physics with a renewed look.

This interrelation operates via the consideration of "critical details", i.e. apparently "small" aspects of student's comments, of the methods and of the taught theory that echo each other. Of course, practical activities proposed to students raise as well such a combined reflection. This will constitute the third example of the paper.

This paper will be concluded by some remarks on the implications concerning teacher acceptance of the innovative, research-based sequences, and teacher training.

Examples of interrelated "What" and "How": looking more closely at Newton's laws

Newton's second law stipulates that the total force exerted by "the exterior" on an object of mass m is linked with the acceleration of the centre of mass of this object, G, by the following simple relationship:

$$\sum \vec{F} = m_T \, \vec{a}_G$$

(in fact, the third law is hidden in this relationship, as it is used to demonstrate that the internal forces have a zero sum).

Students' ideas on this topic have raised innumerable studies, which mainly stress a common trend to linking force and velocity, or more broadly force and motion, instead of force and acceleration. Two aspects have been much less emphasised, and are commented on hereafter.

Common ideas on some aspects of the content: reconsidering the "what" as well as the "how"

An aspect of the second law is the fact that, in order to analyse the motion of the centre of mass G, it is not needed to know the particular point on which the external forces are acting. In this respect, this law is extremely simple and convenient to apply. In order to see if this aspect was well understood, a question has been posed by Menigaux (1994) to students who knew very well this law and were used to bringing it to bear in solving classical problems.

A circular puck can move across a rectangular horizontal frictionless table. At initial time it is situated against one of the edges. A same, constant, force is acting on it, this in two cases. In case (a), the force is applied at the point of the puck which is the nearest to the opposite edge, in case (b), it is exerted on a point of the puck that is initially on the side of the puck.

> *The question is: will the puck take the same time to cross the table and hit the opposite edge, in the two cases, and if not in which case does it take the longer time?*

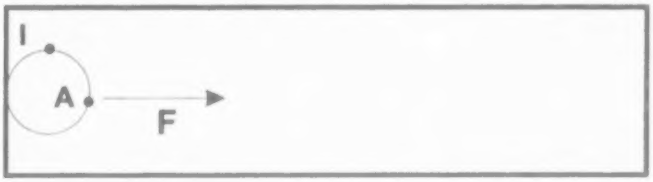

FIGURE 1. A puck on a horizontal frictionless table; a constant force is applied always at the same point, either A (case a) or I (case b).

The correct answer is that it takes the same time in the two cases, because only the total exerted force matters to determine the motion of the centre of mass G, motion which itself determines the travel time. This correct answer relies on the relationship recalled above:

$$\sum \vec{F} = m_T \vec{a}_G$$

The motion of the puck is different in the two cases. In the first case, the motion is rectilinear with a constant acceleration; in the second case, two things are happening at the same time: a rectilinear motion for G (the same as in case (a)) and a pendular rotation of the puck around G. Some studies about students' common ideas permit to predict that case (b) is likely to favour the answers: "longer time, because there is a rotation before the translation occurs", or "longer time, because it takes more time to effectuate these two things at a time"; or else, "the point on which the force is applied travels a longer way in case (b)". Indeed, such answers were found in the consulted students, a result confirmed by a very recent replication (Rigaut 2000). In these two research studies, the salience of the particular point on which the force acts is patent. In particular, the trajectory of this point draws students' attention, and intervenes many times in their justifications.

Paying attention to this common answer leads one to realise that a fundamental aspect of this law is not commonly underlined in teaching. Indeed, the situation used by Menigaux may seem somewhat exotic, but in fact it points to the great power of the above relationship. By using only, in teaching, examples of motion that are pure translations or at best complicated motions of isolated systems, a striking aspect of the theory is left aside, if not hidden. If one decides to fight "wrong ideas" as those investigated by Menigaux, for instance by introducing "apparently complicated" but in fact still simple examples of combined motions, then a new teaching goal is adopted, that is, to "make physics speak", in this case to show that – in a large range of problems – the points where the forces are acting are not relevant (the only thing mattering being if they act or not on the considered object or system). The goal is not only to teach how to avoid a trap and to fight an obstacle, but to express more explicitly, although without calculation, what physics says. The questions of "how" and "what" to teach are interrelated.

Critical details of students' answers: "force of...", and the mixing of Newton's second and third law

As seen above, analysing students' ideas is an opportunity to reconsider not only our "methods" but also the taught matter and possible teaching goals in terms of content. Such a reflection may be raised by apparently small aspects of students' comments.

A critical detail in learners' common way of speaking is the expression "force of ". Instead of mentioning, as explained above, the force exerted on an object, learners often say for instance that a mass "has" some upward force that explains its upward motion, or comment that the force "of" an object being larger, its velocity is larger. This may go with the wish of finding a cause to a motion, whereas the required cause is anterior to the considered time: storing the cause into the mobile may be seen as helpful. Scholars use as well, or at least do not pay attention to this kind of expression. Commenting a dialogue during which a pupil said: "I don't think the table has a force", McDermott (1984, p 145) wrote "the idea that the table could exert a force was generally not accepted by the children" with no special comment about the difference between "to have", and to "exert" a force. In the Berkeley Physics Course (vol. 3, p 86) it is said that a mass "had" a return force (per...) to mean that a force was exerted on it (by a spring). These examples are not isolated. Often, the weight of an object is said to "act (as well) on its support", or the centrifugal force "of" a ball in circular motion at the end of a string is seen as acting on the ball itself (in the frame of reference of the room). But there is a risk of confusing, under the shelter of this "of", the object that exerts the force with the object on which it is applied. This improper expression coincides with an anthropomorphic style, and might contribute to blurring the distinction that should be done between Newton's second and third law.

The third law, indeed, puts into play two distinct objects or systems, A and B. It is worth recalling that the well known relationship $\boldsymbol{F}$ (A on B) = $-\boldsymbol{F}$ (B on A) holds whatever the motions of A and B may be. For example, when a body suspended from a spring is accelerating upwards, the action of the body on the spring is still exactly opposite to the reaction of the spring on the mass. But an anthropomorphic view of such a situation leads to seeing it as a dynamical conflict between two objects in which the strongest of them determines a global motion in the direction of its own effort: "the spring imposes its forces to the body". Then, the common link between (total) force and observed motion applies with an "$\boldsymbol{F}$" which is the "total" force in a balance implying the mutual forces between two distinct objects (other forces being forgotten). A common conclusion is that "the reaction of the spring is larger than the action of the body" or "the third law does not hold anymore". In the same way, a driver (A) pushing its car (B) on a flat road towards a garage, or a nail (A) hammered into a board (B), are very commonly said to violate the third law, A exerting on B a force larger than the reciprocal.

Rather paradoxically, situations with objects in contact are still more difficult because it is tempting to compare the forces acting on the contact point, point seen as an object in itself, as if the second law was to be used here.

One of the facets of students' difficulties being to mistake forces acting on distinct objects for forces acting on the same object, it is necessary to look for some means of clarifying which force is acting on what. Verbal language should be unambiguous but the correct expression "Force exerted by A on B" easily becomes "Force of A on B", which meets with the "Force of" denounced above, ascribed to the object itself as an anthropomorphic feature. This verbal closeness weakens the clarifying power of verbal statements. Common drawings of objects in contact, with mutual forces drawn on the contact point, are also improper to evacuate the usual confusions.

This is why it seems useful to draw the forces in a more appropriate way, objects being represented with a space between them even if they are actually in contact (Fig. 2b). A different coding for each interaction, the compulsory mentioning of two forces per interaction -gravitation included – are additional aids which require students to perceive, for instance, that a body with a mass M does not "exert its weight on the spring", or that the Archimedes' interaction also results in a downwards push of the immersed body in the liquid. In such a mapping, equality between intensities – at rest (Fig. 2c) – does not mean an indistinction. The weight of the body is explicitly "married" with another force exerted on the earth; and its action on the spring, though of same intensity, is married with another force, the force exerted by the spring on the body.

In the end, when stressing this method, a new way of looking at the content is adopted: less emphasis on detailed motion, kinematics of point mass, point on which a force is applied, more on identifying which object a force is acting on, which partners are interacting, distinguishing properly the second and the third law, unifying the description of interactions (Archimedes' interaction is an ordinary interaction: there are reciprocal forces), utilising the hard core of the laws. Actually, doing this, it seems as if new teaching goals were aimed at. Such goals have actually inspired the syllabus for grade 11 in France in 1992.

To sum up, being aware of the importance of a "detail" in students' comment "force of..", attentive to the content (the second and the third laws are not identical), and in search of an efficient technique to represent forces, are interrelated concerns. Each of these attitudes is linked to the others, and, in the present case, is not straightforward.

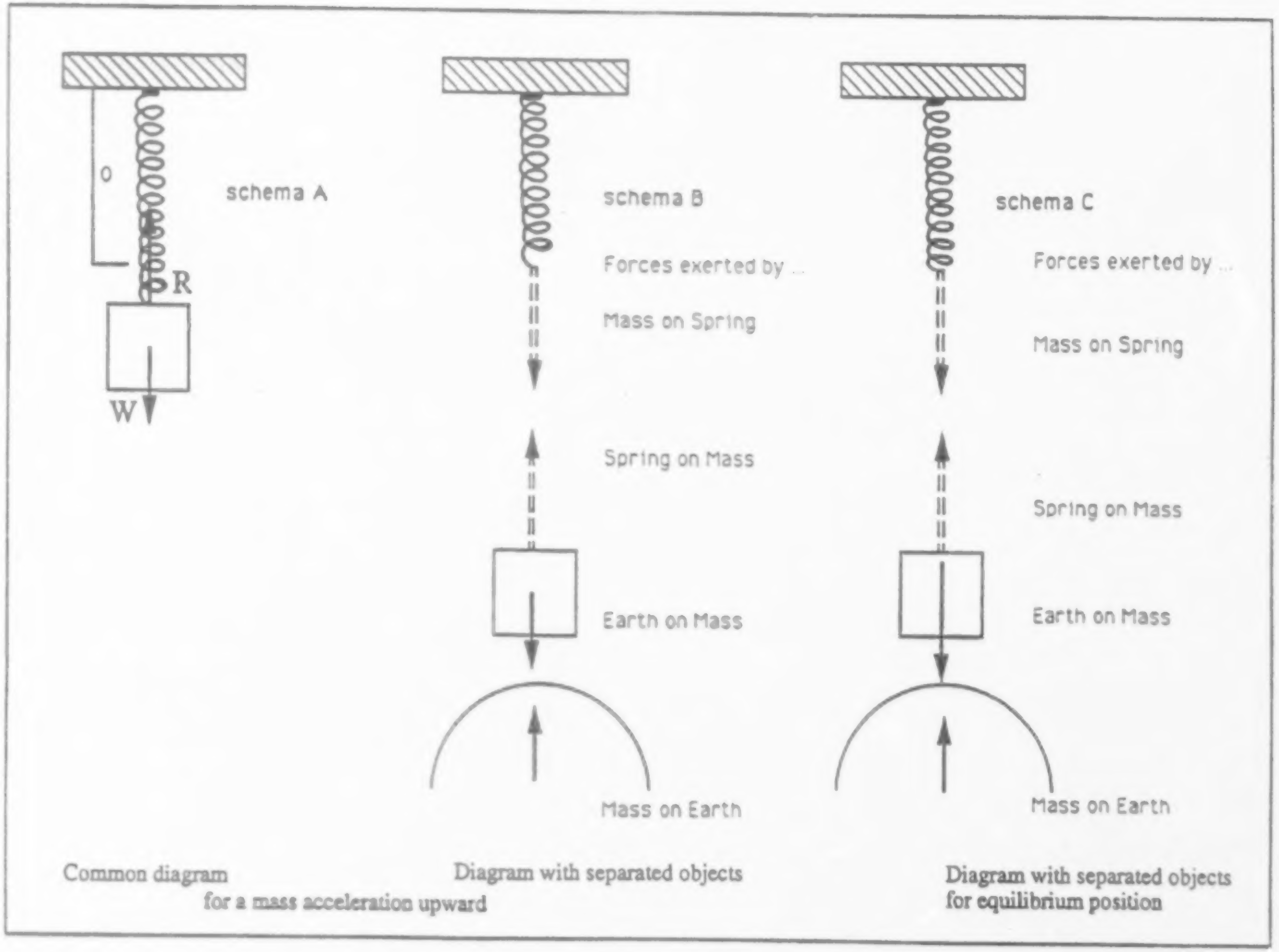
schema A
0
R
W
Common diagram
for a mass acceleration upward
schema B
Forces exerted by ...
Mass on Spring
Spring on Mass
Earth on Mass
Mass on Earth
Diagram with separated objects
schema C
Forces exerted by ...
Mass on Spring
Spring on Mass
Earth on Mass
Mass on Earth
Diagram with separated objects
for equilibrium position

FIGURE 2. Two different ways of representing forces (a: common; b and c: with separated objects).

Practical activities, details and content: an example about colour

The next example will show that methods and content analysis may be simultaneously reconsidered. An innovative sequence has been designed by Chauvet about colour, on the basis of an investigation about common ideas on this topic. It can be briefly described as follows.

This sequence aims to clarify the concept of colour for teachers. It stresses the fact that colour is a perceived response to received light (for more detail, see Chauvet 1996a and b). A structure is proposed to analyse all phenomena of colour and to guide students' reasoning about colour: a chain of transformation of information carried by light to the observer's eye (Fig. 3).

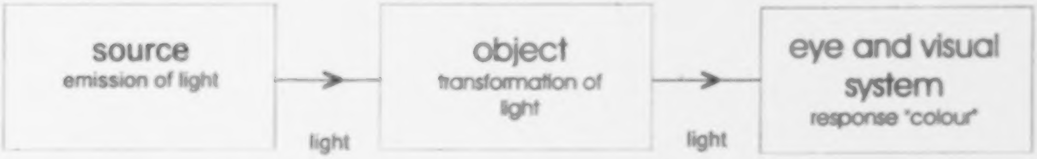

FIGURE 3. "Chain": analysing successive changes in information about colour carried by light to the eye.

The sequencing of taught concepts is not a traditional one. The classical spectral analysis is postponed after two phases of teaching. The first phase consists of destabilising students by adding coloured lights (red, green, and blue), which produces colours that are different from those obtained by mixing paints. In the second phase, devoted to absorption, a situation of coloured shadows is analysed: when an obstacle blocks off one of the coloured lights that, when added, constitute white light, the lack of a spectral band results in colour. The part played by absorption is stressed and shown to be similar for filters and pigments. Additive and subtractive syntheses are analysed within the same frame of interpretation: the chain of events occurring to light, and the response of visual system.

An overall feature is that a particular emphasis is put on learning cycles involving prediction, experiment and debate. One of the practical activities envisaged in this sequence concerns the colour of coloured letters lit in coloured lights, on a black background. Then, a difficulty occurs, which provides us with an instructive example.

A red letter lit in blue should, in principle, appear as black, because as a red filter (third of spectra), it should absorb a blue light, the sole light received in this case. In fact, it appears dark violet. This coincides with a prediction inspired by a wrong idea. Indeed, students commonly consider that lighting a coloured object with coloured light is like mixing paints, and, if understood as paints, red + blue gives violet: the observed colour. Would experimental facts betray correct physics and favour wrong ideas? This misleading effect is due to experimental problems (there is some ambient light in the classroom, including some red radiation, and the red paper does not diffuse a spectrum with a sharp bandwidth, which results in violet diffused light of very weak intensity).

What is recommended by Chauvet is to focus students' observation not on the hue of the observed letters, but on their visibility. This means that attention is drawn on the amount of diffused light. This is very convincing, because in this respect the prediction based on correct physics coincides with what is observed: the red letter lit in blue light is hardly visible on a dark background, which witnesses the strong absorption which constitutes the first order effect. This is also in line with the conceptual backbone of the sequence: the "chain" of events occurring to light, at the expense of juggling with rules about hues.

This example is typical of the combined effort that is necessary in didactic research here concerning the part of experiments in teaching: attention to details, be they "practical", and, given that experiments do not speak by themselves, specification of how observation should be guided and debate-oriented, in close relationship to the particular aspects of the content the understanding of which is aimed at. This effort changes critically the outcome of teaching, as shown by Chauvet.

It is an understatement to say that this is not obvious. For instance the idea of questioning students about the visibility and not the hue of the coloured letters does not emerge naturally from teaching practice. It is even difficult for trained teachers to realise its importance.

The wish to "show" is always to be tightly framed by a thorough didactic analysis of the conceptual aspects involved. Another, less subtle, example is that of the "materialised rays" so much in favour to "demonstrate" rectilinear propagation of light, and which, if not very cautiously presented and discussed, are likely to reinforce the idea that light is visible in itself: what to say about "materialised rays" that, due to a light source placed behind a slit on a diffusing paper, are therefore not in the same plane as the source?

Critical details: a problematic take-up by teachers

Two recent research studies (Hirn & Viennot 2000; Chauvet submitted) concern the teaching of elementary optics, and bear on the way teachers, when implementing research-based teaching sequences, transform the didactic intentions of the innovators, a theme more widely developed in the STTIS programme (see STTIS 1998, 1999 a and b).

The findings indicate that some aspects of the innovative sequences are willingly adopted, such as a new theme – vision – or, still more easily, a new device – like punched screens or a small white tetrahedron lit in lights of wide bandwidth (a third of the spectrum of white light). But what appears as much more difficult is the take-up of details that are critical to ensure the conceptual consistency and the didactic efficiency of the sequence. For instance, Hirn shows that the way teachers use "materialised rays" in their teaching is quasi unanimously classical, that is as a demonstration of the rectilinear propagation at the beginning of the sequence, the presence of dust being mentioned without explaining the path of the light to the observers' eye. Or else, contrary to what is recommended in official texts, teachers without special training do not link, in their explanation, the brightness of some areas of a screen and what can be seen from behind a hole punched in each of these areas. Rather, they put separate questions on these two aspects. Similarly, Chauvet underlines that the global organisation of her sequence about colour, as well as the suggested material devices, are reasonably well accepted, whereas some specific conceptual nodes are neglected. Thus, the recommended way of conducting debates about coloured letters lit in coloured lights (see above) is quasi unanimously "forgotten" by the consulted teachers.

Conclusion

This discussion and the experimental facts recalled above may appear as discouraging. Obviously, didactics is not simply a matter of arranging series of concepts in new syllabuses, it cannot either be conducted only on the basis of broad, although indispensable – guidelines such as "take students' ideas into account", or "foster scientific debate in the classroom". Most probably, it is in their detailed implementation that such principles play their actual relevance. This meets with what Millar (1989) was recommending, i.e. designing teaching sequences at the micro-level. One may regret this state of affairs, the overall difficulty, the lack of universal recipies. But there are positive aspects.

One is that physics – chemistry, biology, and math – also becomes more interesting when "pushed into the corners". It is because it says things that are different from common sense that we learn something from physics. And it is by examining in detail what is said in both registers that the differences appear. In this sense, a training in didactics fosters a better knowledge of physics. Didactics does not invent a new physics, but may lead one to consider differently a given corpus of knowledge, in particular by stressing some points that are often occulted.

Another good reason to go on with optimism is that, when research-based sequences associate carefully designed strategies with precise teachings goals, a great attention being given to (many) apparently small aspects, the success is impressively stable, even if limited. The sequence by Chauvet is an example of such a success as regards the trained students (Chauvet 1996 a and b; see also, for a sequence about electrostatics, Viennot & Rainson 1999). It is also very effective in terms of teacher training. For instance, the trained teachers actually take-up the recommended management of practical activities, asking for a reasoned prediction before experimenting, then for debates, whereas it is observed in other cases (STTIS 1999b), for instance in Hirn's study, that teachers massively ignore this type of recommendation. Another clue is that teachers' attitude with respect to images concerning colour is impressively changed after such a training.

The relative failure of teacher training which is often observed (Gil-Perez & Pessoa de Carvalho 1998) might be partly due to an approach of the required changes which is either too general, aiming for instance at changing teachers' views of science, or too technical, i.e. focused on the take-up of a given sequence. The interrelation between the desirable components of a didactic sequence need to be illustrated at the micro-level in order to convince teachers of the importance of each one.

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The Force Concept Inventory in Diagnosing the Conceptual Understanding of Newtonian Mechanics in Finnish Upper Secondary Schools

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A project has been started at the University of Helsinki in the Department of Physics, which aims at the assessment of learning results of introductory mechanics in Finnish upper secondary schools. Conceptual understanding of mechanics is evaluated by using the Finnish translation of the Force Concept Inventory (FCI) developed at the Arizona State University. FCI is one of the most used conceptual tests. It is intended to determine whether students are able to distinguish between correct Newtonian answers and popular but erroneous common-sense beliefs. The first results in Finnish upper secondary schools are reported and the results are compared with the results from the USA.

Background

During the last years much emphasis has been paid to the improvement of physics instruction in Finland. The programmes of the government of Finland in 1995–99 and 1999 onward started efforts to raise the level of skills in science and mathematics in Finnish schools (LUMA 1999). Experimentality (demonstrations, practical work, and investigations) is seen in a central role of science education and it is believed to improve learning results. In the latest national guidelines for school curriculum design, prepared by the Finnish National Board of Education, the idea of experimental approach in science teaching was addressed and recommended (NBE 1994). However, there have been very few quantitative instruments for evaluating instruction in Finland. Therefore the Force Concept Inventory (FCI) (Hestenes et al. 1992) has been translated into Finnish to be used for evaluating physics instruction.

The purpose of the study

The purpose of the study presented here is to test the applicability of the translated FCI in evaluating student performance in Newtonian mechanics in Finnish upper secondary schools.
The research questions are:

1. What is the average level of conceptual understanding of Newtonian mechanics in Finnish upper secondary schools measured with the translated FCI?
2. How common are the common-sense conceptions, especially the impetus and dominance conceptions, among Finnish pupils?
3. How do the Finnish results compare with the results obtained in the USA?

The present study is part of a more extensive research, which aims at investigating teachers' conceptions of the role of experimentality and the concept of interaction in instruction of mechanics, and among all clarifying how these conceptions affect students' conceptual understanding of Newtonian mechanics.

Common-sense conceptions

Physics education research has shown the important role of students' common-sense conceptions based on everyday experiences, in learning (e.g. Driver 1983; Pfundt & Duit 1993). In order to be effective instruction has to take these conceptions into account. Especially it has turned out that the common-sense conceptions concerning force and motion are incompatible with

Newtonian mechanics. These conceptions are firmly established and it is very difficult to produce change in them (Duit 1993; McDermott 1998).

The two most frequent common-sense conceptions in mechanics are impetus and dominance (Halloun et al. 1985a). According to impetus conception a force has to be exerted on an object in order to keep it in motion. An object can acquire impetus from a hit and the motion stops when the impetus wears out, as if the force is bound in the object as a kind of internal force. This conception is inconsistent with the Newton's first law. According to the dominance principle a greater object exerts a greater force on the other in an interaction. This conception is in turn inconsistent with the Newton's third law (e.g. Viiri 1995).

Multiple-choice diagnostics in surveying students' common-sense conceptions

Multiple-choice diagnostics have become a popular instrument to evaluate physics instruction and assess students' common-sense beliefs. One of the most used and best-known diagnostics is the Force Concept Inventory (FCI) (Hestenes et al. 1992). The test is based on research results on students' common-sense conceptions about motion (Halloun et al. 1985a). The FCI was designed to improve the Mechanics Diagnostic test described in Halloun et al. 1985b. The validity and the reliability of the test have been carefully established e.g. by using interviews. Because the questions are essentially the same the validation for the Mechanics Diagnostics is valid for the FCI as well (Hestenes et al. 1992).

The diagnostics consists of 30 multiple-choice, qualitative questions about mechanics. The choices have been designed so that the student has to decide between a common-sense and a Newtonian conception. Below is an example of a question of the renewed version of the FCI (published in e.g. Mazur 1997).

A boy throws a steel ball straight up. Consider the motion of the ball only after it has left the boy's hand but before it touches the ground, and assume that forces exerted by the air are negligible. For these conditions, the force(s) acting on the ball is (are)

a) a downward force of gravity along with a steadily decreasing upward force.
b) a steadily degreasing upward force from the moment it leaves the boy's hand until it reaches the its highest point; on the way down there is a steadily increasing downward force of gravity as the ball gets closer to the Earth.
c) an almost constant downward force of gravity along with an upward force that steadily decreases until the ball reaches its highest point; on the way down there is only an almost constant downward force of gravity.
d) an almost constant downward force of gravity only.
e) none of the above. The ball falls back to ground because of its natural tendency to rest on the surface of the Earth.

BOX 1. An example of an item of the FCI.

The Force Concept Inventory probes for students' ability to apply basic concepts and laws of Newtonian mechanics in various contexts. The test ignores problem-solving skills. It should be pointed out that the FCI-results give only a very restricted view on the understanding and mastering of the Newtonian mechanics, but it makes the comparison of the results possible. Despite of its restrictions the results of the test have proved to be useful in improving instruction (Wells et al. 1995).

Methods and material of the present study

Since the FCI provides a means to probe for the frequency of occurrence of common-sense beliefs among students participated in the research, it was chosen as the diagnostic method. The FCI was chosen also in order to be able to compare the results in Finland with international ones. The sample consisted of almost 400 students from different parts of Finland. The results to be obtained here can thus be generalised concerning understanding of basic concepts of mechanics in Finnish upper secondary schools.

The translation was done carefully. First, the researchers became familiar with the research in the field of students' conceptions in mechanics. Second, the researchers discussed with the authors of the original FCI to confirm of the meaning and background of different alternatives. Third, three researchers translated the FCI separately and then compared these translations, from which the final translation was derived. Fourth, 15 university students read and answered the FCI and gave feedback about translated version. Consequently, the translation can be considered to be adequate for this study.

The research was conducted during 1999–2000 in Finnish upper secondary schools. 18 teachers from 18 different schools participated as volunteers in the research. The FCI was given to 386 students after mechanics instruction, which concluded the basics kinematics, dynamics and the Newton's laws. The teachers were given the test just before testing, so it was not possible to teach to the test. The results were analysed with the SPSS-programme.

Results

Means and distributions of the total score

The mean score of the FCI test is 17.4 (maximum 30) and the standard deviation 6.3. The distribution of the total scores is displayed in Figure 1.

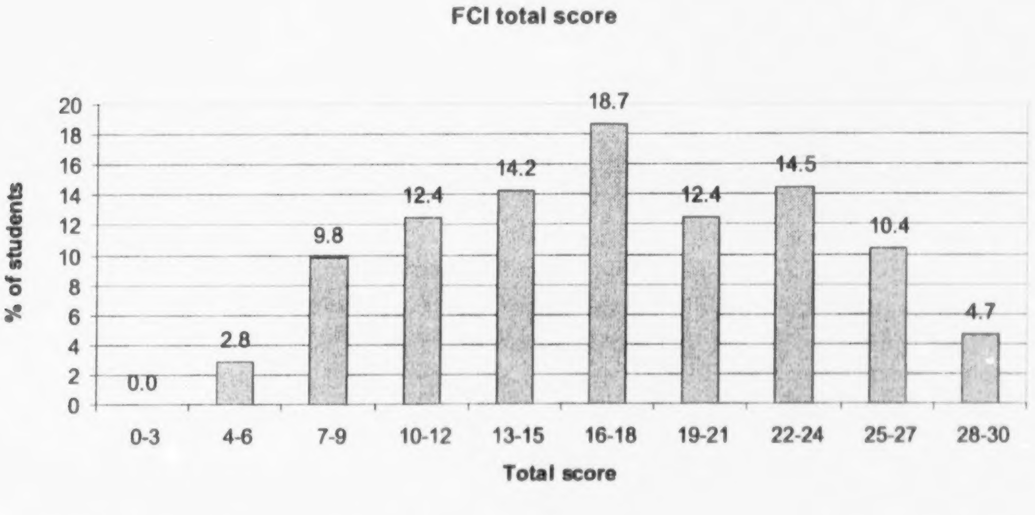

FIGURE 1. Distribution of the total scores.

The designers of the test have proposed an idea of the Newtonian conceptual threshold, that is a minimum level of understanding the Newtonian mechanics, to be 60% of the maximum score (Hestenes et al. 1992). In this study, 52% of students score below this threshold. Students whose total score is at least 80% of the maximum can be regarded as Newtonian thinkers (Hestenes et al. 1992). Only 20% of the students in this study reach this level of understanding of Newtonian mechanics.

29% of the students are girls. Their mean score is 15.3 (s. dev. 5.9), while the mean score of boys is 18.3 (s. dev. 6.2). The difference is statistically significant (t=4.4**).

Scores on kinematics

The understanding of the concepts of position, velocity and acceleration is probed by two questions in the test. 48% of the students had difficulties in these questions. 16% of the students answered in both questions incorrectly. These results suggest that about half of the students have difficulties in understanding the concepts of position, velocity and acceleration, which are an essential foundation for the Newtonian force concept.

Scores on items revealing the conceptions of impetus

According to the impetus belief an object receives, for example in a hit, an internal force, which keeps the object in motion. Three items were especially designed to detect that belief. In those items a situation where a ball was thrown or a puck was hit were described. The question was about the forces acting on the object after the hit or throw. The choices with a force in the direction of the motion were considered as expressions of the impetus belief. Figure 2 represents the number of choices according to the impetus belief. 36% of the students answered according to the impetus belief consistently in all the three items. 19% answered in two and 12% in one item according to the impetus belief. 27% of the students chose all alternatives according to Newtonian mechanics. The Newtonian choices are indicated in the following figures with a dark column. In addition, 6% chose an alternative according to some other conception.

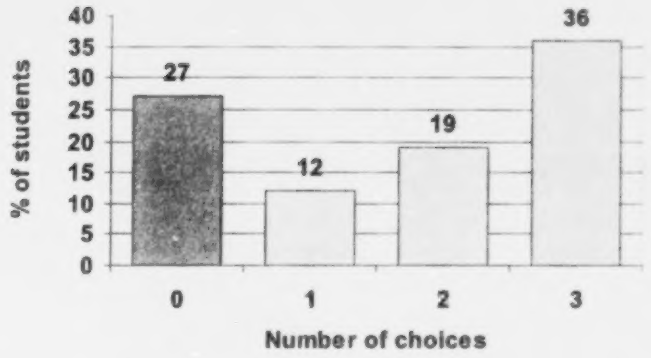

FIGURE 2. Distribution of the number of incorrect choices for items measuring the impetus belief (3 items altogether).

As figure 2 shows, the majority of the students answered consistently either according to the impetus belief or to the Newtonian concept.

According to the Newton's Second Law constant force implies constant acceleration. Four items were designed to probe for the ability to apply this law in various contexts. Only 9% of the students answered correctly to all these items. Figure 3 shows how students were uncertain in their choices and chose sometimes according to Newtonian mechanics and sometimes according to a common-sense conception.

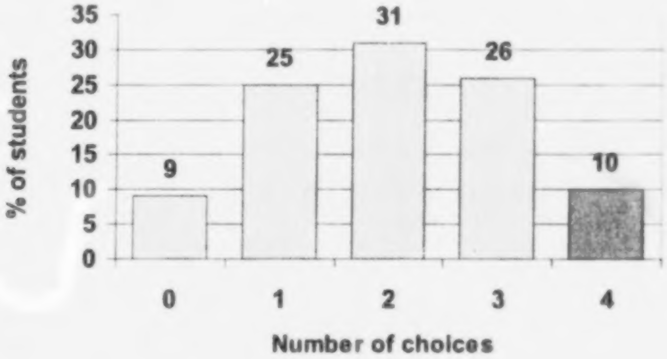

FIGURE 3. Distribution of the number of correct choices for items measuring the mastering of Newton's Second law (4 items altogether).

Scores on items revealing the conception of dominance

Two basic questions were designed in the test to probe for the dominance principle. In the first a situation is described where a large truck collides with a small car, in the second two students are sitting on office chairs. One student has his feet on the knees of the other student and pushes outward with his feet. In both items it is asked to compare the forces, which the parties exert on each other. In figure 4 the number of choices according to the dominance conception is displayed. About half of the students answered according to the Newton's Third Law in both items. Whereas, 22% of the students systematically answered according to the dominance principle. According to them the bigger or more active party exerts a greater force on the other.

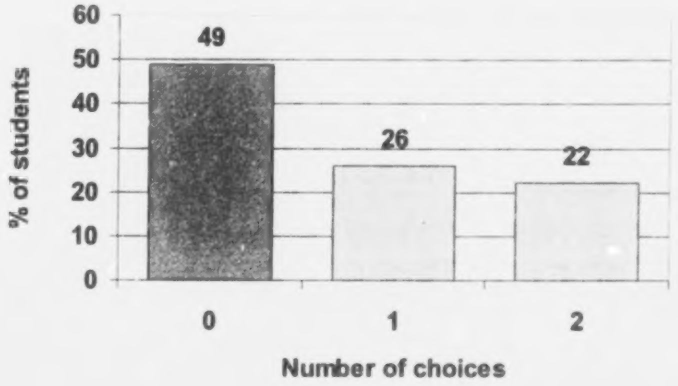

FIGURE 4. Distribution of the number of incorrect choices for items measuring the dominance principle (3 items altogether).

The understanding of the Newton's Third Law in a more complicated context was probed by two other questions in addition to the above-discussed ones. In these items a system in motion with constant velocity or constant acceleration is considered. It turns out that 92% of those who answered correctly to the first two items answer also correctly according to Newton's Third Law in the case where the system is in constant motion. However, only 59% of them mastered the symmetry of the interaction also in a system with constant acceleration. In all, only 28% of the students mastered the Newton's third law in all various contexts.

The differences in results between schools are of no importance for the present study. This comparison will be reported in an other study, where the connection with teachers' conceptions of the role of experimentality and learning results is discussed in more detail.

Comparison of the Finnish results with the results from the USA

The Force Concept Inventory has widely been used to evaluate physics instruction in the USA. During the last decade data has been amassed on 20 000 high school students. In the USA the average scores after traditional (teacher-centred) instruction is 42% and 2/3 of the students fail to reach the Newtonian conceptual threshold of 60%. Students of teachers, who have fully adopted the Modelling Method, a research based teaching methodology (e.g. Hestenes 1996) in physics instruction reach the average score of 69%. (Popp 2000)

In this study the average score of the Finnish students is 58%. That is higher than the average score after traditional physics instruction in the USA but lower than the average score after instruction according to the Modelling Method. The more accurate comparison of the Finnish results with the results from the USA was therefore chosen to be done with data containing both teachers using traditional methods and Modelling Methodology. The respective American sample consists of 2118 students of 34 teachers all over the USA. 24 of the teachers reported using the Modelling Method, whereas 10 used mostly traditional methods. The mean score is 57%. The data is collected during 1994–99. (Popp 2000)

The distributions of the chosen alternatives in each item were compared with the chi-square test. In three items, where 20% of the expected values were less than 5 or at least one expected value was zero, the Yates correction was used in order to determine the value for chi-square. The chi-square test shows that in 8 items out of 30 there is no statistical difference in the distributions of the chosen alternatives between Finnish and American students. For example the distribution of the item 29 ($\chi^2 = 8{,}0^{ns}$) is displayed in Figure 5. The question is displayed in Box 2. The alternative 2 is the correct one.

An empty office chair is at rest on a floor. Consider the following forces:

A. A downward force of gravity.
B. An upward force exerted by the floor.
C. A net downward force exerted by the air.

Which of the forces is (are) acting on the office chair?

1. A only.
2. A and B.
3. B and C.
4. A, B and C.
5. None of the forces. (Since the chair is at rest, there are no forces acting upon it.)

BOX 2. Item 29.

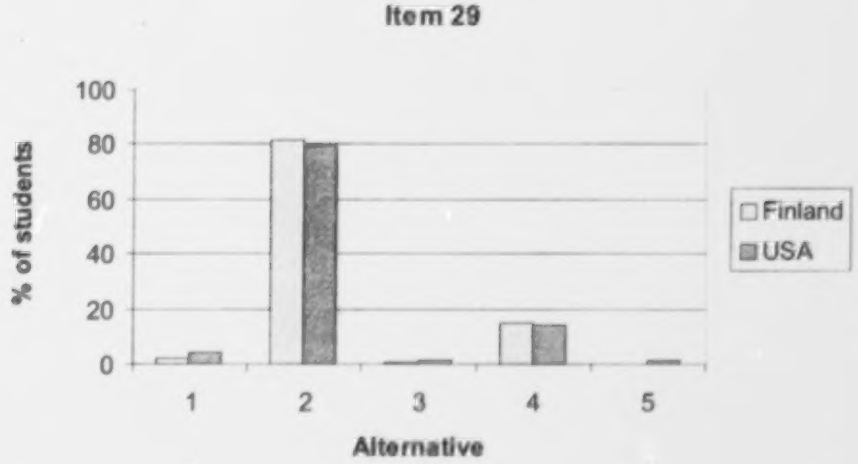

FIGURE 5. Distributions of the chosen alternatives in item 29.

In 25 items out of 30 the chi-square is below 50. For example in item 25 displayed in Box 3 and Figure 6. ($\chi^2 = 49.8^{**}$). The alternative 3 is the correct one.

A woman exerts a constant horizontal force on a large box. As a result, the box moves across a horizontal floor at a constant speed v_0.

The constant horizontal force applied by the woman

1. Has the same magnitude as the weight of the box.
2. Is greater than the weight of the box.
3. Has the same magnitude as the total force that resists the motion of the box.
4. is greater than the total force that resists the motion of the box.
5. Is greater than either the weight of the box or the total force that resists its motion.

BOX 3. Item 25.

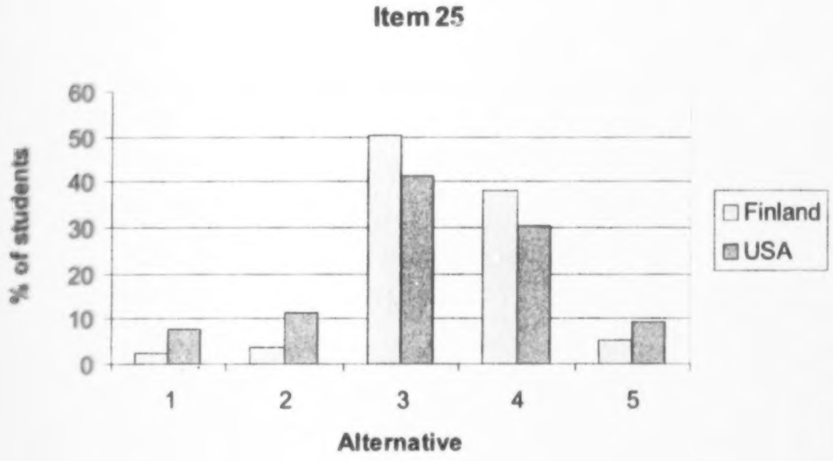

FIGURE 6. Distributions of the chosen alternatives in item 25.

Although the statistical analysis of the distributions of the chosen alternatives shows a difference between the Finnish and the American students, the Figure 6 indicates that students in both countries choose the same alternatives even though not with exactly equal frequency. The chi-square was greatest for the item 13 ($\chi^2 = 156.2^{**}$). The question has been displayed earlier in

Box 1. The distribution of the chosen alternatives is shown in Figure 7. The alternative 4 is the correct one.

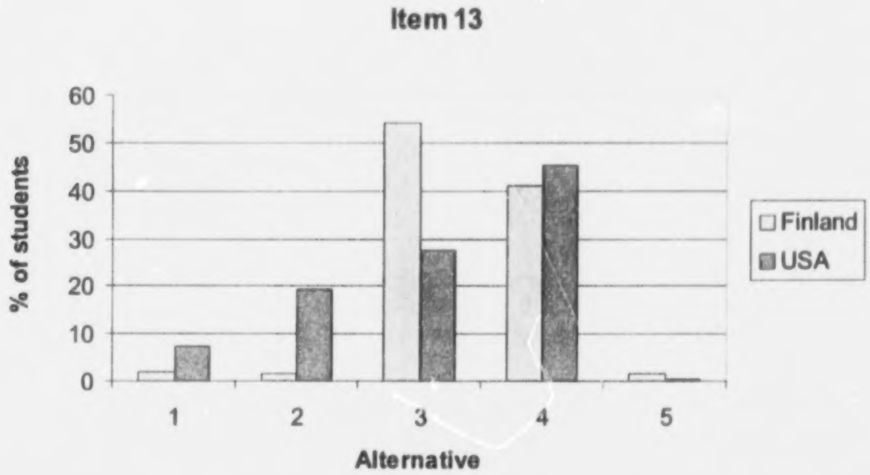

FIGURE 7. Distributions of the chosen alternatives in item 13.

The figure 7 shows that the choices of Finnish students are concentrated on the 3. and 4. alternatives, while the American students quite frequently choose also the 2. alternative. This suggests that Finnish students have less the conception that the downward force of gravity increases significantly when the ball falls down. It is unlikely that this difference of the distributions is due to the translation. Also in the other 4 questions where the chi-square is high and the distributions differ from each other the students in both countries choose the same alternatives although not quite with the same frequency.

The statistical analysis shows that the distributions of the chosen alternatives are largely the same for the Finnish and the American students. This means that the students choose the same alternatives in Finland and in the USA. Which in turn indicates that the students in both countries have the same common-sense conceptions.

Had there been large deviations in the distributions in the chosen alternatives between the Finnish and American pupils, there could have been reason to revise the translation. Because this was not the case, there were no items which are either rejected or accepted with much larger frequency than in the USA, we assume that the translation does not lead to anomalies due to apparent hints of correct or incorrect answers contained in the wording or way of posing the questions (Hestenes 2000).

Conclusions

In this study the average level of conceptual understanding of Newtonian mechanics of 386 students in Finnish upper secondary schools has been examined during 1999–2000 using the Finnish translation of the Force Concept Inventory. The results suggest that only 48% of students reach the minimum level of understanding of the Newtonian mechanics during the mechanics instruction. Only 20% of the students can systematically apply the Newtonian concepts in various contexts.

The results presented are consistent with the results obtained by Viiri (1995). Viiri has used part of the Force Concept Inventory in evaluating a teaching experiment at a technical institute. The mean of the test, which contained 14 items, was 58%, which is exactly the same as the mean of the complete test of the students participating in this study.

Results obtained by the Finnish translation of FCI were compared with the results from the USA. The translation of the test, which is also available as a web version, can be used in physics education research. The results presented in this report can be used as a reference in evaluating the effectiveness of teaching experiments. In addition the translation is a valuable diagnostic tool for teachers in recognising students' common-sense conceptions before instruction and in improving instruction.

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An Evaluation of Interactive Teaching Methods in Mechanics: Using the FCI to Monitor Student Learning

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This paper addresses issues relating to the teaching and learning of Newtonian mechanics in a Finnish High School (student aged 16–19). The impact of interactive teaching methods on student learning was investigated using the Force Concept Inventory (FCI). The work presented here is part of the preliminary phase of an ongoing research program in which the author acts as teacher and researcher. The results indicate that the teaching approach is very promising in enhancing conceptual understanding in mechanics.

Introduction

This paper reports part of an on-going research program on teaching for conceptual understanding and conceptual coherence in high school physics. The research is directed towards a doctoral thesis in physics education but professional development is also a goal. High school refers to the Finnish "lukio" and International Baccalaureate (IB) programs. Completing high school in Finland usually takes three years (age 16–19). The author teaches physics in both programs in Kuopio Lyseo High School, and carries out research at the same time.

The research was motivated by the results that a group of students in the preliminary IB year (age 16) achieved in the Force Concept Inventory (FCI) a few years ago. It was an eye-opener: the students had not learnt much in terms of conceptual understanding. The outcome was disappointing and made me wonder why only a few students scored well. The teaching then consisted of traditional lecturing and demonstrations. Students were asked questions involving conceptual understanding but only the most active students actually participated in the lessons; the majority was silent and spent their time writing down lecture notes. I realized that something very crucial was missing from the teaching. Interestingly, Mazur (1997) realized the very same thing after giving the FCI to his students in Harvard University for the first time.

As a result of his experience Mazur (1997) developed an interactive teaching method called *Peer Instruction* to improve students' conceptual understanding in an introductory university physics course. I decided to adopt Peer Instruction for two reasons: firstly, Mazur provided evidence that there actually was a significant change in learning after using Peer Instruction, and secondly, Mazur's method seemed to be adaptable to a high school setting. I integrated several other research-based methods into Mazur's Peer Instruction. The instructional approach developed is described in detail in the next section.

Mazur had used the FCI to evaluate Peer Instruction (Crouch & Mazur 2001). The FCI is a systematically developed and validated multiple choice tests (Hestenes et al. 1992a). It has gained a lot of popularity in assessing introductory mechanics courses in high schools, colleges and universities in the USA. The teaching approach I developed and used has also been evaluated in terms of student gains in the FCI. It was not possible to use an experimental design in this study for two reasons. Firstly, a rigorous sampling of students into control and experiment groups is very hard to implement in the Finnish high school system. Secondly, the experimental approach, in which one would teach the control group using conventional methods, would be ethically difficult to justify because I am convinced that the developed teaching approach is better than the lecturing method I employed earlier. Using other teachers in teaching control groups is problematic as well, for the reasons discussed by Leach and Scott (2000).

Interactive Conceptual Instruction

The teaching approach I used – Interactive Conceptual Instruction (ICI) – can be divided into three components, which overlap with each other to some extent:

- Conceptual focus
- Classroom interactions and research-based materials
- Use of texts

Conceptual focus

A considerable amount of attention is devoted to developing students' conceptual understanding before they start quantitative problem solving. Conceptual focus is achieved by utilising the principle of "concepts first" (van Heuvelen 1991; Gautreau & Novemsky 1997). New ideas are first developed at a conceptual level with little or no mathematics (definitions are usually expressed in mathematical form). Graphical and diagrammatic representations are introduced at an early stage as well; they are used in conjunction with verbal explanations. Only after the students have a good grasp of concepts is quantitative problem solving introduced. An understanding of concepts as they are applied to empirical situations is developed through simple demonstrations and collaborative pair discussion. The students do not, however, "discover" concepts by themselves. The teacher carefully stages teaching activities, the term "staging" referring to how a teaching activity is presented and mediated by the teacher (Leach and Scott 2000).

Classroom interactions and research-based materials

"Classroom interactions" mean employing collaborative pair discussions and teacher talk in making meaning (Scott 1998). Peer Instruction, as developed by Mazur at Harvard University, is used to actively engage the students in the learning process. Mazur (1997) describes the method thus:

"The basic goals of *Peer Instruction* are to exploit student interaction during lectures and focus students' attention on underlying concepts. Instead of presenting the level of detail covered in the textbook or lecture notes, lectures consist of a number of short presentations on key points; each followed by a

ConcepTest—short conceptual questions on the subject being discussed. The students are first given time to formulate answers and then asked to discuss their answers with each other. This process (a) forces the students to think through the arguments being developed, and (b) provides them (as well as the teacher) with a way to assess their understanding of the concept."

I have modified Mazur's Peer Instruction to some extent (Savinainen 1999, 2000a, 2000b and 2001). The students discuss conceptual exercises in pairs in the way described by Mazur. It is very important that after pair discussion students compare their explanations with the explanation provided by the teacher. Demonstrations and laboratory experiments serve as an excellent source of conceptual questions, and points arising from them are used as a basis for oral conceptual exercises. This has advantages and disadvantages. The advantage is that conceptual questions can be flexibly tailored to match the teaching situation at hand. The disadvantage is that these exercises are not so carefully crafted as research-based materials.

In addition to oral questions I use research-based materials to promote understanding and address any difficulties that students may have. Appropriate multiple choice questions (e.g. *ConcepTests*, Mazur 1997) and conceptual exercises developed through physics education research are used. The Ranking Task Exercises (O'Kuma et al. 2000), for instance, are very useful. These exercises involve multiple representations: conceptual, diagrammatic, graphical, and mathematical. Both school physics and everyday contexts are used in the conceptual exercises. For example a rollerblade video (Etkina 1998 and 2000) offers very good examples of an everyday context. I have used the video to illustrate Newton's laws in a virtually frictionless setting, using conceptual questions, which the students discuss in pairs.

Use of texts

I use texts in various ways to promote learning. The students do not take ordinary notes: instead, they make additions, remarks and underlinings in the textbook. They do not have to waste time and attention in writing something that is in the book. This approach is strongly advocated by Mazur (1996) also. I often ask the students to read the textbook beforehand: this releases time for active discussion. Conceptual change takes much more time than teachers usually realize (Harrison et al. 1999). However, giving up time-consuming notetaking makes it possible to teach almost as many topics as before. Traditional notes are replaced by *concept maps*, which I have constructed. Concept maps allow the students to see "the big picture" and relations of key ideas in a concise form (Novak & Gowin 1984). Explicit references are made to the concept maps throughout the teaching. Many students have found the concept maps an efficient tool for revision (Savinainen 2000a and 2000b). In addition, the students are encouraged to write their own summaries.

Problem solving can be used to clarify concepts as well. Problems that have multiple steps and make use of many physical principles are especially useful for *Strategy Writing* (Leonard, Dufresen & Mestre 1996). In this approach physical principles and assumptions are carefully written down before calculations are worked out. It helps the students to connect conceptual and mathematical representations in a meaningful way. However, Strategy Writing has only a supplemental nature in the teaching approach taken here: few exercises are treated in the course using Strategy Writing. This is because it takes quite a lot of time.

Students' opinions on the teaching approach have been sought via a questionnaire, and the responses have been very positive in general. Most students have stated that peer discussions and emphasis on concepts have made learning physics easier, and some students have voluntarily added that the method has made studying even enjoyable!

Example lesson

An example lesson is presented in Table 1. The lesson introduces the concept of acceleration to students aged 16, and lasts 45 minutes.

TABLE 1. An example lesson on mechanics, introducing acceleration. The students read the chapter on acceleration in the textbook before the lesson.

Activity	Time	Comments
Checking homework on velocity (problem solving and conceptual exercises from the book)	10 min	Some problems are presented on a white board. Conceptual homework exercises are first discussed in pairs.
Teacher explains the idea of change in volocity.	5 min	It is explained that magnitude or direction (or both) of velocity can change.
Short demonstrations in which the teacher moves in different ways. Students discuss in pairs if the teacher is accelerating or not.	10 min	Demonstrations are used as a source of peer discussion questions. The questions are posed orally.
Teacher explains uniform acceleration.	3 min	A falling marker is used as an example.
Students are asked to present uniform motion and uniform acceleration using velocity against time graphs. The graphs are constructed and discussed in pairs.	7 min	Uniform motion and the graphical determination of average velocity were introduced using data measured by the students.
Velocity against time graphs are presented on the whiteboard. Students explain and execute the motion in pairs.	10 min	Graphical exercises will be continued in the next lesson in which students empirically determine acceleration of a falling object using ticker timer tapes.
Conceptual and graphical homework exercises and/or reading tasks are set for the next lesson.		No calculations are carried out in the introduction lesson. Calculations will follow the qualitative treatment.

The Force Concept Inventory

The FCI is a multiple choice test designed to measure students' understanding of force and related kinematics concepts (Hestenes et al. 1992a). It is used as an assessment tool at every level of introductory physics instruction from high schools to universities, especially in the USA. It is often given as a pre-test at the beginning of a course and then as a post-test at the end.

The FCI has a precursor: the Mechanics Diagnostic Test (MDT; Halloun & Hestenes 1985). The MDT questions were designed around common misconceptions of the force concept. The test was initially given in open-answer form to introductory level college students. Then a multiple-choice version was created, which contained the most common open-ended answers as choices. Various versions of the test were administered to more than 1000 college students. The reliability and validity of the MDT were established using interviews and statistical analysis. Kuder-Richardson reliability coefficients were determined to be 0.86 for pre-test use and 0.89 for post-test use. These high values are indicative of a highly reliable test.

About half of the FCI questions are essentially the same as those in the MDT. The authors of the FCI did not, however, repeat the lengthy procedures to establish test reliability and validity for the FCI. They justified this by conducting interviews and by pointing out that similar scores were found on both tests in similar populations. A Revised Version of the FCI was developed and placed on the web in 1995 (Halloun et al. 1995). It later appeared in Mazur's book (Mazur 1997). The 1995 version has 30 questions, whereas the original FCI had 29 questions.

The main findings reported by Hestenes et al. (1992a) are listed below:

1) Math background is not a major factor in determining high school FCI scores.
2) Pre-test scores are uniformly low for beginning students.
3) Post-test scores exhibited no relationship with students' socio-economical level.
4) No large gains from pre- to post-test were seen with conventional instruction.
5) There was no correlation between post-test scores and teacher competence. (There was one exception: one teacher had a very low score (39%) from the MDT. His students had also the lowest post-test results from the FCI).
6) An FCI score of 60% is the entry threshold to Newtonian physics. Below that limit, "students' grasp of Newtonian concepts is insufficient for effective problem solving".

Later, Halloun and Hestenes (1995) added one more finding:

7) An FCI score of 85% is the Newtonian mastery threshold.

Additional important findings come from Hake's (1998a) large survey study. Hake collected data from 62 introductory level courses involving over 6500 students in the USA, with the aim of comparing MDT or FCI scores with type of instruction. Even though the students were not randomly assigned from a single large homogeneous population to the traditional and interactive-engagement courses, the data are all drawn from the same institutions and the same generic introductory program regimes (Hake 1998b).

Hake (1998a) introduced an average normalized gain as a way to analyze the data. It is the ratio of the actual average gain to the maximum possible average gain:

$$(1) \qquad < g > = \frac{< S_{post} > - < S_{pre} >}{100\% - < S_{pre} >}$$

where $< S_{post} >$ and $< S_{pre} >$ are the final (post) and initial (pre) class averages. The average gain provides a way to compare classes with very different FCI scores because it normalizes scores based on how well the class performed before the instruction. There was a very low correlation (+0.02) of <g> with $< S_{post} >$ for the 62 survey courses. This means that <g> is practically independent of the students' initial scores.

The average normalized gain (<<g>>) of fourteen traditional survey courses (2048 students) was 0.23 ± 0.04sd (Hake 1998a). The traditional courses relied primarily on passive-student lectures, recipe labs and algorithmic-problem exams. Forty-eight interactive-engagement (IE) survey courses yielded <<g>> = 0.48 ± 0.14sd. Hake defined IE methods "as those designed in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors".

Hake (2001a) concluded that the results strongly suggest that

- traditional courses fail to convey much basic conceptual understanding of Newtonian mechanics to the average student.
- IE courses can be much more effective than traditional courses in enhancing conceptual understanding.

On the basis of the data Hake (1998a) classified introductory mechanics courses into the following regions:

1) "High-g" courses: (<g>) > 0.7;
2) "Medium-g" courses: 0.7 > (<g>) > 0.3;
3) "Low-g" courses: (<g>) < 0.3.

No course reached the High-g region (the best <g> was 0.69). All the traditional courses fell into the Low-g region. It implies that gains in *traditional* high school, college and university physics courses were largely independent of the teachers' experience and academic background. This result supports the finding of the authors of the original FCI that no large FCI gains were seen with conventional instruction. The FCI therefore provides a useful instrument to be used in evaluating students' understanding of Newtonian mechanics. Some justification for the use of the American findings in the case of Finnish FCI results will be provided in the next section.

Using the FCI to evaluate student learning

Interactive Conceptual Instruction groups

Table 2 presents FCI results for the Interactive Conceptual Instruction (ICI) groups. A Finnish translation of the 1995 version of the FCI was used (Koponen et al. 2000). The ICI group results are matched, i.e. given for students who took both the pre and post FCI. The correlation between individual students' gains and pre-test results was very close to zero (+0.055). The Kuderson-Richardson reliability coefficient KR20 was 0.86 for the pre-test and 0.84 for the post-test (calculated using the combined ICI group, n = 75). These are in very good agreement with the coefficients reported by the authors of the MDT (precursor of the FCI).

All the ICI groups fall into the Medium-g region defined by Hake. In addition, the ICI1 post-test average was near the Newtonian mastery threshold defined by Halloun and Hestenes (1995). Both Finnish National Curriculum groups (ICI1 and ICI2) had followed two traditional high school courses dealing with mechanics before the Interactive Conceptual Instruction course; this explains a fairly high pre-test result in group ICI1. Interestingly the other

TABLE 2. Pre and post FCI results of the Interactive Conceptual Instruction (ICI) groups. The average normalized gains (<g>) have been calculated using more significant figures than shown in the table. The presented effect size is Cohen's d. Standard deviations are shown in parentheses.

Group	N	Pre-test % (S.Dev.)	Post-test % (S.Dev.)	<g>	Effect size
ICI1[1]	32	60 (19)	81 (14)	0.52	1.2
ICI2[1]	21	44 (13)	70 (18)	0.46	1.6
ICI3[2]	22	28 (14)	69 (17)	0.57	2.6

[1] Finnish National Curriculum students (age 17–18).

[2] Preparatory International Baccalaureate students (age 16).

Finnish group, ICI2, had followed the very same courses but their pre-test result was significantly lower. Another teacher had taught the previous courses and they did not go very deep into the concept of force.

The ICI3 group had received no high school instruction on the concept of force before the interactive introductory course. This is reflected in the very poor pre-FCI result (28%). The probability of guessing correct answers in the FCI questions is 20%. The ICI3 group is quite similar to the preparatory International Baccalaureate group I taught a few years earlier. That group was taught using traditional methods with an emphasis on conceptual understanding. The post-test result of a shortened old FCI (25 questions) was 58%. Even though there is quite a clear difference between post-FCI results (58% vs. 69%), no conclusion can be drawn on this basis because the earlier group did not take a pre-FCI. It must be noted, however, that the results do not exclude the possibility that the change could be partially attributed to Interactive Conceptual Instruction.

Average normalized gains suggest that the results are very promising. Further support for the conclusion is gained from effect size, which is a family of indices that measures the magnitude of a treatment. For this study, a relevant measure of effect size is Cohen's d (Cohen 1988; Becker 2001):

$$d = \frac{< S_{post} > - < S_{pre} >}{\sigma_{pooled}} \tag{2}$$

$$\sigma_{pooled} = \sqrt{\frac{\left(\sigma_{pre}^{2} + \sigma_{post}^{2}\right)}{2}} \tag{3}$$

where $< S_{post} >$ and $< S_{pre} >$ are the final (post) and initial (pre) class averages; σ_{post} and σ_{pre} are the final and initial standard deviations. Cohen (1988) defined effect sizes as "small, d = 0.2; medium, d = 0.5; and large, d = 0.8." Effect sizes for the study groups were well above the high boundary.

Other Finnish groups

As mentioned in the introduction, very few Finnish data have been published on the FCI. Some Finnish FCI results are presented in this section. Of course, these other groups do not constitute control groups but the results provide some information on FCI gains for students having more or less the same pre-course curriculum history as the Interactive Conceptual Instruction groups.

Pre and post FCI results of a Finnish high school group and two university groups are presented in Table 3 (Jauhiainen 2001). The high school group (HS) had traditional teaching, which was complemented by few interactive-engagement exercises. The university group UNI1 consisted of physics students, who followed a traditional introductory course in mechanics, having previously completed high school courses in the subject. The university group UNI2 consisted of students, who studied to become physics, mathematics, and chemistry teachers. These students followed interactive-engagement teaching in didactical physics. The same translation of the 1995 version of the FCI was used as in the Interactive Conceptual Instruction groups.

TABLE 3. Pre and post FCI results of one Finnish high school group (HS) and two groups (UNI1 and UNI2) in a Finnish University.

Group	N (pre)	N (post)	Pre-test %	Post-test %	<g>
HS	25	31	54	65	0.23
UNI1	155	32	71	79	0.26
UNI2	27	24	79	87	0.37

The post-test results shown in Table 3 are respectable but the gains in the traditional groups (HS and UNI1) are in the Low-g region. The results of the UNI1 group are not very informative because the majority of the students did not take a post-test. The UNI2 group (IE teaching) reached the Medium-g region. These results are consistent with Hake's (1998a) findings in American institutes.

Viiri (1995 and 1996) used a shortened version (14 questions) of the original FCI in his thesis on a constructivist teaching experiment in engineering education. Results from his study (denoted by Viiri) are presented in Table 4, together with results from three high school groups (Pehkonen 1999). The groups used the same shortened version of the original FCI as Viiri. Teachers who had participated in special training in didactical physics taught two high school groups (HS1 and HS2), whereas a teacher, who had not been involved in special training, taught one group (HS3). Post teaching results of the ICI groups in the same 14 questions from the 1995 version of the FCI are shown as well.

TABLE 4. Pre- and post-test results of the shortened FCI in various Finnish institutions. The study groups are denoted by *ICI*, other high school groups by *HS* and the engineering group by *Viiri*. Standard deviations are shown in parentheses.

Group	N	Post-test % (S.Dev.)
Viiri	94	58 (22)
HS1	25	48 (21)
HS2	30	61 (21)
HS3	32	30 (18)
ICI1	32	83 (14)
ICI2	21	72 (17)
ICI3	22	68 (17)

All the ICI groups had a greater average in the shortened post FCI than had the other groups. Viiri's post-test result (58%) was better than the result of the comparison group in his study (Viiri 1995, 1996). He compared the FCI responses in his study group with American results reported in Hestenes et al. (1992), and Viiri concluded that the results were very similar. This suggests that physics students have the same difficulties in the USA and Finland in learning the concept of force. Viiri's conclusion and the limited data in Tables 3 and 4 provide some justification for evaluating Finnish FCI results using the findings of American physics education research.

I have used some other American research-based conceptual inventories to evaluate the teaching approach:

- Mechanics Baseline Test (Hestenes et al. 1992b)
- Test of Understanding Graphs in Kinematics (Beichner 1994)
- Force and Motion Conceptual Evaluation (Thornton & Sokoleff 1998)
- Conceptual Survey in Electricity and Magnetism (Maloney et al. 2000)

The results of these tests are consistent with the FCI results of the ICI groups. It is interesting to note that two successive International Baccalaureate groups in Kuopio Lyseo High School (age 17, combined n = 22) had a higher Mechanics Baseline post-test result (71%, s.d. 15%) than any high school or college in Hake's (1998a) survey. The students had received Interactive Conceptual Instruction.

Discussion

The results of this study strongly suggest that Interactive Conceptual Instruction is useful in teaching high school mechanics. All the available comparison data from Finnish high schools and other institutions indicate that the results are very promising. This conclusion receives support from Hake's large survey in American institutions. The normalized gains of the Finnish ICI groups were consistently better than gains achieved by American high schools, colleges or universities which used traditional teaching methods. Additional support is gained from effect sizes, which show that the magnitude of the treatment (teaching) was high. The validity of using Hake's findings as a basis for comparison raises the question: would the survey results be the same if the study were repeated in Finland? There are not enough data at the moment to answer this question but it would be very surprising if very different results were achieved in Finland.

It cannot be concluded, however, that the very promising results are due only to Interactive Conceptual Instruction. The teacher's expertise has a central influence on the effectiveness of any teaching approach. It is possible and quite likely that the gains in learning in this study are partially due to improvements in the teacher's effectiveness in engaging with students' thinking (Leach & Scott 2000). On the other hand, Hake (1998b) convincingly argues that good teachers are necessary but not sufficient for high quality instruction.

There is also a question of possible biases in the results presented. One such worry is teaching to the test. In a sense, there *was* teaching to the test: the very aim of the teaching was to give students some understanding of the force concept. Conceptual understanding was much more emphasized than in the traditional approach, which focuses mainly on problem solving. There was also a deliberate attempt to avoid exercises identical to the test items. However, some overlap is bound to happen; for instance, it is hard to teach projectile motion without discussing thrown objects, which are addressed in some FCI questions. Another possible worry is test-question leakage. This is unlikely in this study because all the test material is always gathered from students. Of course, the students are allowed to see their responses and the correct answers. It is highly unlikely that Finnish students would find the original publications. There is one exception though: the FCI has been available on the World Wide Web. To prevent students from searching the net for it, it is advisable not to use the name "Force Concept Inventory" in test papers (Hake 2001b).

One could question the use of multiple choice tests by claiming that they do not provide detailed information on students' thinking. It must kept in mind that no matter how good an instrument is, it should be supplemented by information from other sources to get a reliable profile of student understanding (Halloun & Hestenes 1996; Steinberg & Sabella 1997). Nevertheless, good multiple choice tests do give a quick and rough estimate of students' progress in terms of conceptual understanding. In addition, the FCI can be analyzed along different conceptual dimensions. The analysis can reveal special difficulties that students may have and give guidance for future teaching. Even examining research-based tests can give a teacher insights, which may be helpful in teaching. The tests may reveal new aspects of the concepts or even correct some misunderstandings held by the teacher. This at least is my experience.

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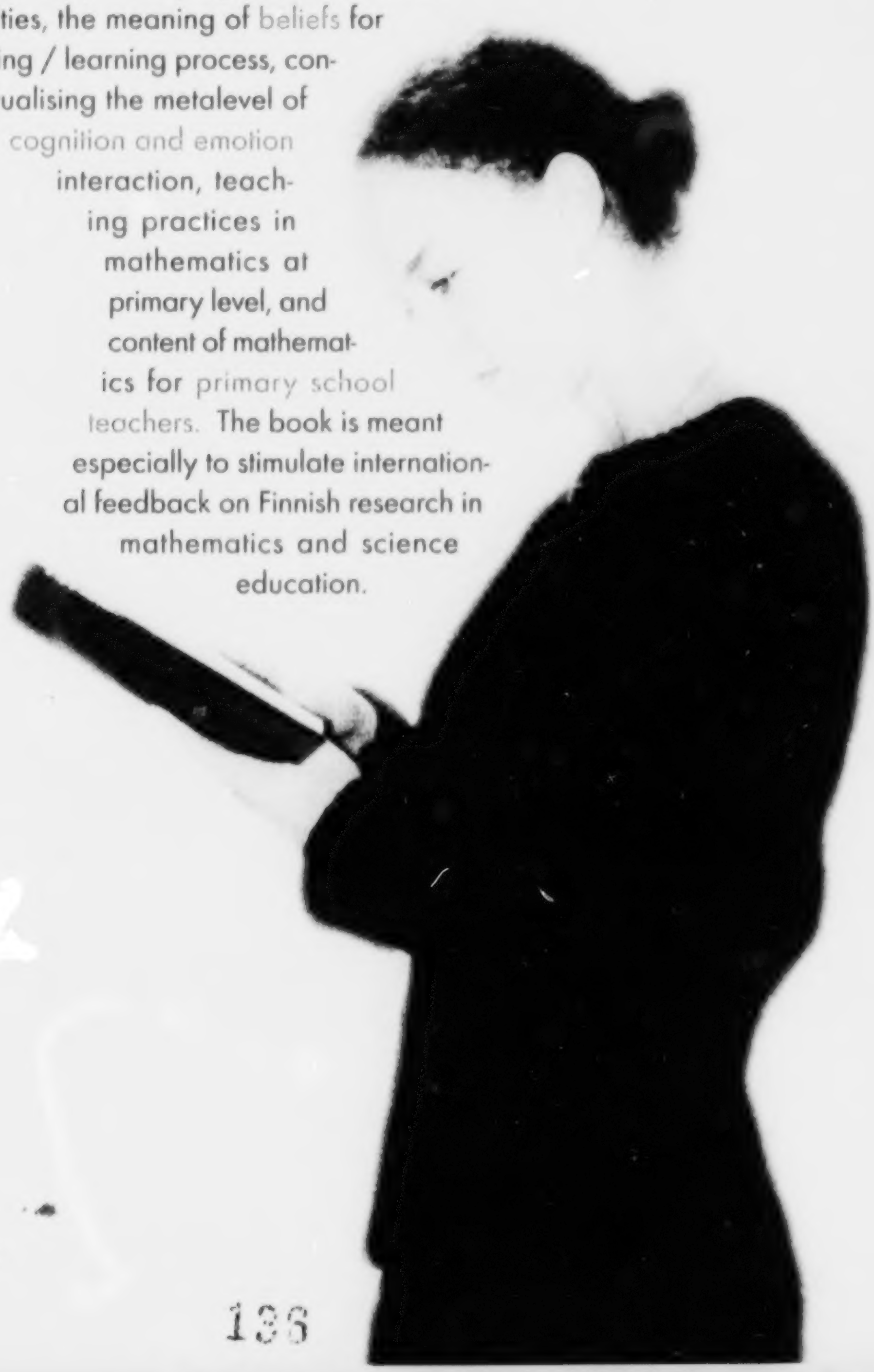
THIS BOOK contains a selection of the papers presented at the seminars of the Finnish Association for Research in Mathematics and Science Education and the Finnish Graduate School of Mathematics, Physics and Chemistry Education. It thus gives a brief insight into some recent research interests in mathematics, physics and chemistry education in Finland. The topics include the role of critical details in finding specific strategies to teach certain contents in physics, interactive teaching methods and conceptual understanding in mechanics, problem solving in chemistry, especially in helping teachers with students' difficulties, the meaning of beliefs for the teaching / learning process, conceptualising the metalevel of cognition and emotion interaction, teaching practices in mathematics at primary level, and content of mathematics for primary school teachers. The book is meant especially to stimulate international feedback on Finnish research in mathematics and science education.



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